Chapter 16: Term Structure of Interest Rates

Solution 16.01

C Section 16.02, Spot Rates

The value of the bond is:

$$\frac{50}{1.05}+\frac{50}{1.06^2}+\frac{1,050}{1.07^3}=\mathbf{949.23}$$

Solution 16.02

E Section 16.02, Spot Rates

We can use the BA-II Plus calculator to answer this question:

50 [÷] 1.05 [+] 50 [÷] 1.06 [x^2] [+] 1,050 [÷] 1.07 [y^x] 3 [=]

(Result is 949.2316)

[+/-] [PV] 3 [N] 50 [PMT] 1,000 [FV]

[CPT] [I/Y]

(Result is 6.9321)

Answer is **6.93%**.

Solution 16.03

C Section 16.02, Spot Rates

Lending annually at 3.0% for 6 years is clearly an inferior strategy, because 3.0% is the lowest possible rate. Likewise lending annually for 3 years at 3.0% and for one 3-year period at 4.0% is clearly inferior to lending for 6 years (two 3-year periods) at 4.0%.

The remaining viable strategies that must be compared are:

Lend for 6 years (two 3-year periods) at 4.0%. This results in an accumulation factor of:

$$\left(1+\frac{0.04}{4}\right)^{24}=1.2697$$

Lend for 1 year at 3.0% and 5 years at 4.4%. This results in an accumulation factor of:

$$\left(1+\frac{0.03}{4}\right)^{4}\left(1+\frac{0.044}{4}\right)^{5\times 4}=1.2823$$

Since the second strategy has a higher accumulation factor, it produces the maximum annual effective rate, which is:

$$1.2823^{1/6}-1=\mathbf{4.23\%}$$

Solution 16.04

B Section 16.02, Spot Rates

The price of a zero-coupon bond as a percentage of its redemption value is equal to the inverse of the accumulation factor achieved by investing in the bond.

Therefore investing X in the 5-month bond, for example, results in an accumulated value of:

$$\frac{X}{0.95}$$

Investing X in each of the bonds results in an accumulated value of:

$$\frac{X}{0.95}+\frac{X}{0.96}+\frac{X}{0.97}+\frac{X}{0.98}+\frac{X}{0.99}$$

$$=X\left[\frac{1}{0.95}+\frac{1}{0.96}+\frac{1}{0.97}+\frac{1}{0.98}+\frac{1}{0.99}\right]=5.1557X$$

Setting this accumulated value equal to 10,000 allows us to solve for X:

$$5.1557X=10,000$$

$$X=\mathbf{1,939.59}$$

Solution 16.05

B Section 16.03, Forward Rates

The spot rates can be used to calculate the forward rate:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_1=\frac{(1+s_2)^2}{1+s_1}$$

$$1+f_1=\frac{1.085^2}{1.075}$$

$$f_1=\mathbf{9.51\%}$$

Solution 16.06

D Section 16.03, Forward Rates

We are being asked for f_2:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_2=\frac{(1+s_3)^3}{(1+s_2)^2}$$

$$1+f_2=\frac{\frac{1}{0.85}}{\frac{1}{0.92}}$$

$$f_2=\mathbf{8.24\%}$$

Solution 16.07

E Section 16.03, Forward Rates

The question is asking for the rate that applies from time 5 to time 6.

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_5=\frac{1.095^6}{1.09^5}$$

$$f_5=\mathbf{12.03\%}$$

Solution 16.08

E Section 16.03, Forward Rates

The loan will be made at the interest rate that can be locked in to apply from time 3 to time 4. This is the 1-year forward rate, deferred for 3 years:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_3=\frac{1.085^4}{1.08^3}$$

$$f_3=0.1001$$

The accumulated value of the loan at time 4 is:

$$1{,}000\times1.1001=\mathbf{1{,}100.14}$$

Solution 16.09

D Section 16.03, Forward Rates

The loan will be accumulated from time 3 to time 4 and from time 4 to time 5.

The 1-year forward rate, deferred for 3 years is found below:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_3=\frac{1.085^4}{1.08^3}$$

$$f_3=0.1001$$

The 1-year forward rate, deferred for 4 years is found below:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_4=\frac{1.090^5}{1.085^4}$$

$$f_4=0.1102$$

The accumulated value of the loan at time 5 is:

$$1{,}000\times1.1001\times1.1102=\mathbf{1{,}221.41}$$

Solution 16.10

B Section 16.03, Forward Rates

The 3-year accumulation factor calculated with the spot rate is the same as the 3-year accumulation factor calculated with the forward rates:

$$(1+s_t)^t = (1+f_0)(1+f_1)\cdots(1+f_{t-1})$$

$$(1+s_3)^3 = (1+f_0)(1+f_1)(1+f_2)$$

$$(1+s_3)^3 = 1.05\times 1.03\times 1.02$$

$$s_3 = \mathbf{3.33\%}$$

Solution 16.11

A Section 16.03, Yield, Spot, and Forward Rates

I. True. When the price of a bond is equal to its par value, there is no premium or discount to compensate for the coupon rate being greater than or less than the yield. Therefore, the coupon is equal to the yield.

II. False. If the spot rates used to discount the cash flows occurring before time n are greater than the n-year spot rate, then the yield on the n-year bond will be greater than the n-year spot rate.

III. False. If the $(n - 1)$-year spot rate is greater than the 1-year forward rate deferred for $(n - 1)$ years, then the n-year spot rate will be greater than the 1-year forward rate deferred for $(n - 1)$ years.

Solution 16.12

C Section 16.03, Forward Rates

The forward rate j applies from time 4 to time 5:

$$j = f_{4,1} = f_4 = \frac{(1+s_5)^5}{(1+s_4)^4} - 1 = \frac{(1+0.02-0.001\times 5+0.001\times 25)^5}{(1+0.02-0.001\times 4+0.001\times 16)^4} - 1$$

$$= \frac{1.040^5}{1.032^4} - 1 = \mathbf{7.26\%}$$

Solution 16.13

C Section 16.02, Spot Rates

The payments occur at times 2, 3, and 4. The present value at time 0 of the annuity is:

$$PV_0 = \frac{10,000}{1.035^2} + \frac{10,000}{1.039^3} + \frac{10,000}{1.044^4} = 26,668.5525$$

The present value at time 0 can be accumulated for one year using the one-year spot rate to find the current value of the annuity in one year:

$$CV_1 = 26,668.5525\times 1.03 = \mathbf{27,468.61}$$

Solution 16.14

A Section 16.03, Forward Rates

Bond B must produce a cash flow at time 2 that is sufficient to grow to 5,000 at a 5.5% interest rate:

$$\frac{5,000}{1.055} = 4,739.3365$$

The combined prices of Bond A and Bond B are:

$$\frac{3,000}{1.04}+\frac{4,739.3365}{1.05^2}=\mathbf{7,183.33}$$

Chapter 17: Interest Rate Swaps

Solution 17.01

D Section 17.01, Establishing the Swap Rate

The swap rate is equal to the coupon rate of a bond that is priced at par. The formula for the coupon rate of a par bond is derived below:

$$F = c \times F \times a_{\overline{n|}spots} + Fv_{spot}^{n}$$

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}}$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - 1.07^{-5}}{\frac{1}{1.03} + \frac{1}{1.04^2} + \frac{1}{1.05^3} + \frac{1}{1.06^4} + \frac{1}{1.07^5}} = \frac{0.2870}{4.2643} = \mathbf{0.0673}$$

Solution 17.02

A Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}}$$

We could use the forward rates to calculate the spot rates, but since the present values calculated with spot rates will be equal to the present values calculated with the corresponding forward rates, we do not need to calculate the spot rates. Instead, we can use the forward rates to find the present values in the formula above. The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - \frac{1}{1.02 \times 1.03 \times 1.035}}{\frac{1}{1.02} + \frac{1}{1.02 \times 1.03} + \frac{1}{1.02 \times 1.03 \times 1.035}} = \frac{0.0804}{2.8519} = \mathbf{0.0282}$$

Solution 17.03

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}}$$

We could use the prices of the zero-coupon bonds to calculate the spot rates, but since the present values calculated with spot rates will be equal to the present values calculated with the prices of the zero-coupon bonds, we do not need to calculate the spot rates. Instead, we can use the prices of the zero-coupon bonds to find the present values in the formula above.

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - 0.93}{0.98 + 0.96 + 0.93} = \frac{0.07}{2.87} = \mathbf{0.0244}$$

Solution 17.04

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}}$$

The price of the zero-coupon bond can be used to find the value of v_{spot}^8:

$$58{,}200.91 = 100{,}000 v_{spot}^8$$

$$v_{spot}^8 = 0.5820091$$

The price of the 3% bond can be used to find the present value of the annuity immediate:

$$76{,}777.78 = 3{,}000 \times a_{\overline{8}|spots} + 100{,}000 v_{spot}^8$$

$$76{,}777.78 = 3{,}000 \times a_{\overline{8}|spots} + 58{,}200.91$$

$$a_{\overline{8}|spots} = 6.1923$$

The swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}} = \frac{1 - 0.5820091}{6.1923} = \mathbf{0.0675}$$

Solution 17.05

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}}$$

The yield of the zero-coupon bond can be used to find the value of v_{spot}^8:

$$v_{spot}^8 = \frac{1}{1.07^8} = 0.5820$$

The yield of the 3% bond can be used to find the price of the bond:

$$P = 3{,}000 \times a_{\overline{8}|6.87\%} + 100{,}000 v^8 = 3{,}000 \times \frac{1 - 1.0687^{-8}}{0.0687} + \frac{100{,}000}{1.0687^8}$$

$$= 76{,}774.2017$$

The price of the 3% bond can be used to find the present value of the annuity immediate:

$$76{,}774.2017 = 3{,}000 \times a_{\overline{8}|spots} + 100{,}000 v_{spot}^8$$

$$76{,}774.2017 = 3{,}000 \times a_{\overline{8}|spots} + 58{,}200.91$$

$$a_{\overline{8}|spots} = 6.1911$$

The swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}} = \frac{1 - 0.5820091}{6.1911} = \mathbf{0.0675}$$

Solution 17.06

C Section 17.01, Establishing the Swap Rate

We begin by finding the 1-year spot rate:

$$s_1 = \frac{103{,}000}{99{,}516.91} - 1 = 0.0350$$

The 1-year spot rate can be used to find the 2-year spot rate:

$$94{,}802.83 = \frac{2{,}000}{1.0350} + \frac{102{,}000}{(1+s_2)^2}$$

$$s_2 = 0.0480$$

The 1-year and 2-year spot rates can be used to find the 3-year spot rate:

$$129{,}505.11 = \frac{15{,}000}{1.0350} + \frac{15{,}000}{(1.0480)^2} + \frac{115{,}000}{(1+s_3)^3}$$

$$s_3 = 0.0430$$

The swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}} = \frac{1 - 1.0430^{-3}}{\frac{1}{1.0350} + \frac{1}{1.0480^2} + \frac{1}{1.0430^3}} = \frac{0.1187}{2.7580} = \mathbf{0.0430}$$

Alternatively, we can save a little time by finding the present value factors instead of the spot rates:

$$v_{spot}^1 = \frac{99{,}516.91}{103{,}000} = 0.9662$$

$$94{,}802.83 = 2{,}000 \times 0.9662 + 102{,}000 v_{spot}^2 \quad \Rightarrow \quad v_{spot}^2 = 0.9105$$

$$129{,}505.11 = 15{,}000 \times 0.9662 + 15{,}000 \times 0.9105 + 115{,}000 v_{spot}^3$$

$$\Rightarrow \quad v_{spot}^3 = 0.8813$$

The swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}} = \frac{1 - v_{spot}^3}{v_{spot}^1 + v_{spot}^2 + v_{spot}^3} = \frac{1 - 0.8813}{0.9662 + 0.9105 + 0.8813} = \frac{0.1187}{2.7580}$$

$$= \mathbf{0.0430}$$

Solution 17.07

C Section 17.01, Establishing the Swap Rate

Since the swap rate is expressed as a quarterly effective interest rate, let's use one quarter as our unit of time.

The swap rate is:

$$c = \frac{1 - v_{spot}^n}{a_{\overline{n}|spots}} = \frac{1 - v_{spot}^4}{v_{spot}^1 + v_{spot}^2 + v_{spot}^3 + v_{spot}^4} = \frac{1 - 0.94}{0.99 + 0.97 + 0.95 + 0.94} = \frac{0.06}{3.85}$$

$$= \mathbf{0.0156}$$

Solution 17.08

E Section 17.01, Establishing the Swap Rate

The swap rate is equal to the coupon rate of a bond that is priced at par. The formula for the coupon rate of a par bond is derived below:

$$F = c \times F \times a_{\overline{n}|spots} + Fv_{spot}^{n}$$

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = \mathbf{0.0444}$$

Solution 17.09

E Section 17.01, Interest Rate Swap Payment

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = 0.0444497$$

Since the swap rate is greater than the floating rate, the fixed-rate payer makes a payment to the floating-rate payer:

$$\text{Net Payment to floating-rate payer} = (0.0444497 - 0.04) \times 100{,}000 = \mathbf{444.97}$$

Solution 17.10

A Section 17.02, Interest Rate Swap Payment

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = 0.0444497$$

The coupon of the hypothetical fixed-rate bond is the swap rate times the notional amount of the swap:

$$c \times F = 0.0444497 \times 100{,}000 = 4{,}444.97$$

At the end of one year, the value of the swap to the fixed rate payer is the value of the hypothetical floating-rate bond minus the value of the hypothetical fixed-rate bond:

$$\text{Value to fixed-rate payer} = F - PV_t(\text{Fixed rate bond})$$

$$= 100{,}000 - \left(\frac{4{,}444.97}{1.04} + \frac{4{,}444.97}{1.05^2} + \frac{104{,}444.97}{1.06^3}\right) = \mathbf{4{,}000.27}$$