CAS/CIA Exam 3
Spring 2007 Solutions

Note: The CAS/CIA Spring 2007 Exam 3 contained 16 questions that were relevant for SOA Exam MFE. The solutions to those questions are included in this document.

Solution 3

B Chapter 1, Put-Call Parity

We are asked to find the arbitrage profit "per share," but it is not clear whether the question means that one share is involved at the outset or that one share is involved at the end of 6 months. Fortunately, both interpretations lead to the same answer.

We can use put-call parity to understand this problem:

$$C_{Eur}(K,T)+Ke^{-rT}=S_0e^{-\delta T}+P_{Eur}(K,T)$$

Put-call parity tells us that the following strategies produce the same cash flow at the end of 6 months:

- Purchase 1 call option and lend the present value of the \$50 strike price. The cost of this strategy is:

$$C_{Eur}(K,T)+Ke^{-rT}=2+50e^{-0.03(0.5)}=51.2556$$

- Purchase $e^{-0.02(0.5)}$ shares of stock and purchase 1 put option. The cost of this strategy is:

$$S_0e^{-\delta T}+P_{Eur}(K,T)=49.70e^{-0.02(0.5)}+2.35=51.5555$$

Since the first strategy costs less than the second strategy, arbitrage profits can be earned by going long the first strategy and shorting the second strategy (buy low, sell high). The net payoff in 6 months is zero, and the arbitrage profit now is:

$$51.5555-51.2556=0.2999$$

Based on this result, Choice B is the correct answer.

Note that the answer above is based on one share at the end of 6 months, but at the outset, only $e^{-0.02(0.5)}$ shares are sold. If we want to determine the arbitrage profits based on the sale of one share at the outset, then we must gross up the strategies above by $e^{0.02(0.5)}$. This results in an arbitrage profit of:

$$0.2999\times e^{0.02(0.5)}=0.3029$$

The correct answer remains B.

Solution 4

E Chapter 1, Put-Call Parity

We can use put-call parity to find the value of the put option:

$$C_{Eur}(K,T)+Ke^{-rT}=S_0-PV_{0,T}(Div)+P_{Eur}(K,T)$$
$$2+30e^{-0.10(0.5)}=29-0.5e^{-0.10(2/12)}-0.5e^{-0.10(5/12)}+P_{Eur}(30,0.5)$$
$$P_{Eur}(30,0.5)=2.5082$$

Solution 12

A Chapter 2, Factors Affecting Premiums

Statement I is true, because an American option provides all of the rights provided by a European option and more.

Statement II is false, because a higher stock price produces a lower payoff for a put option.

Statement III is false, because a higher strike price reduces the payoff of a call option.

Solution 13

A Chapter 1, Synthetic T-bills

We can use put-call parity to find the interest rate:

$$C_{Eur}(K,T)+Ke^{-rT}=S_0+P_{Eur}(K,T)$$
$$6.70+80e^{-r\times 0.25}=85+1.60$$
$$e^{-r\times 0.25}=0.99875$$
$$-r\times 0.25=\ln(0.99875)$$
$$r=0.005$$

Solution 14

D Chapter 3, Two-Period Binomial Model

The risk-neutral probability of an upward move is constant since:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.05-0.00)1} - \frac{90}{100}}{\frac{110}{100} - \frac{90}{100}} = 0.75636$$

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.05-0.00)1} - \frac{99}{110}}{\frac{121}{110} - \frac{99}{110}} = 0.75636$$

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.05-0.00)1} - \frac{81}{90}}{\frac{99}{90} - \frac{81}{90}} = 0.75636$$

If the final stock price is \$121, then the payoff of the call option is \$121 – \$95 = \$26. If the final stock price is \$99, then the payoff of the call option is \$99 – \$95 = \$4. If the final stock price is \$81, then the payoff of the call option is \$0. The value of the call option is:

$$V(S_0,K,0) = e^{-r(hn)} \sum_{j=0}^{n} \left[\binom{n}{j} (p^*)^j (1-p^*)^{n-j} V(S_0 u^j d^{n-j}, K, hn) \right]$$

$$= e^{-0.05(2)}[(0.75636)^2(26) + 2(0.75636)(1-0.75636)4 + 0]$$

$$= 14.7924$$

Solution 15

C Chapter 3, Risk-neutral Probability

The values of u and d are:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.05-0.035)(1/3)+0.30\sqrt{1/3}} = 1.19507$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.05-0.035)(1/3)-0.30\sqrt{1/3}} = 0.84518$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.05-0.035)(1/3)} - 0.84518}{1.19507 - 0.84518} = 0.4568$$

Solution 16

C Chapter 3, Replication

The option pays 6 in the up-state and 0 in the down-state:

$$V_u = Max[0, 18 - 12] = 6$$
$$V_d = Max[0, 4 - 12] = 0$$

The dividend rate is zero, so the number of shares needed to replicate the option is:

$$\Delta = e^{-\delta h} \frac{V_u - V_d}{S(u-d)} = e^{-0 \times 1} \frac{6 - 0}{10(18/10 - 4/10)} = 0.42857$$

Solution 17

D Chapter 3, Two-Period Binomial Model

Since no dividend is mentioned, we assume that the dividend rate is zero.

The values of u and d are:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.05-0.00)(1) + 0.35\sqrt{1}} = 1.49182$$
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.05-0.00)(1) - 0.35\sqrt{1}} = 0.74082$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.05-0.00)(1)} - 0.74082}{1.49182 - 0.74082} = 0.41338$$

The stock price tree and its corresponding tree of option prices are:

Stock			European Put		
		77.8939			0.0000
	52.2139			0.0000	
35.0000		38.6810	3.9830		0.0000
	25.9286			7.1378	
		19.2084			12.7916

The put option can be exercised only if the stock price falls to \$19.2084. At that point, the put option has a payoff of \$12.7916. Although the entire table of put prices is filled in above, we can directly find the value of the put option as follows:

$$V(S_0, K, 0) = e^{-r(hn)} \sum_{j=0}^{n} \left[\binom{n}{j} (p^*)^j (1-p^*)^{n-j} V(S_0 u^j d^{n-j}, K, hn) \right]$$
$$= e^{-0.05(1)(2)} \left[0 + 0 + (1 - 0.41338)^2 (12.7916) \right] = 3.9830$$

Solution 20

B Chapter 7, Black-Scholes Call Price

The Black-Scholes Formula uses the risk-free interest rate, not the expected annual return.

The first step is to calculate d_1 and d_2:

$$d_1 = \frac{\ln(S/K)+(r-\delta+0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(58.96/60)+(0.06-0.05+0.5\times0.20^2)\times0.25}{0.20\sqrt{0.25}}$$
$$= -0.09985$$
$$d_2 = d_1 - \sigma\sqrt{T} = -0.09985 - 0.20\sqrt{0.25} = -0.19985$$

We have:

$$N(d_1) = N(-0.09985) \approx N(-0.10) = 1 - N(0.10) = 1 - 0.5398 = 0.4602$$

$$N(d_2) = N(-0.19985) \approx N(-0.20) = 1 - N(0.20) = 1 - 0.5793 = 0.4207$$

The value of the European call option is:

$$C_{Eur} = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$$
$$= 58.96e^{-0.05(0.25)} \times 0.4602 - 60e^{-0.06(0.25)} \times 0.4207 = 1.9301$$

Solution 21

A Chapter 7, Black-Scholes Put Price

Since no dividend is mentioned, we assume that the dividend rate is zero.

The first step is to calculate d_1 and d_2:

$$d_1 = \frac{\ln(S/K)+(r-\delta+0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(9.67/8.75)+(0.08-0.00+0.5\times0.40^2)\times0.25}{0.40\sqrt{0.25}}$$
$$= 0.69987$$
$$d_2 = d_1 - \sigma\sqrt{T} = 0.69987 - 0.40\sqrt{0.25} = 0.49987$$

We have:

$$N(d_1) = N(0.69987) \approx N(0.70) = 0.7580$$

$$N(d_2) = N(0.49987) \approx N(0.50) = 0.6915$$

The value of the European put option is:

$$P_{Eur} = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$
$$= 8.75e^{-0.08(0.25)} \times (1-0.6915) - 9.67e^{-0.00(0.25)} \times (1-0.7580)$$
$$= 0.3058$$

Solution 32

C Chapter 9, Delta-Hedging

We are not told the interval of time over which the stock price increases. Instead, we are told that the stock price "quickly jumps." We can take this to mean that the stock price increase is instantaneous. Therefore there is no need to consider the impact from borrowing or lending at the risk-free rate.

The delta of a call option covering 1 share is:

$$\Delta_{Call} = e^{-\delta T} N(d_1) = e^{-0.07\times 1} 0.5793 = 0.540136$$

The market-maker sold 100 call options, each of which covers 100 shares. Therefore, from the perspective of the market-maker, the delta of the position to be hedged is:

$$-100 \times 100 \times 0.540136 = -5,401.36$$

To delta-hedge the position, the market-maker purchases 5,401.36 shares of stock. When the stock price increases, the market-maker has a gain on the stock and a loss on the call options:

Gain on Stock:	$5,401.36 \times (41-40) = 5,401.36$
Gain on Calls:	$-100 \times 56.08 = \underline{-5,608.00}$
Net Profit:	-206.64

The market-maker has a net loss of 206.64.

Solution 33

D Chapter 9, Market-Maker's Profit

If a market-maker writes an option and delta-hedges the position, then the market-maker's profit is:

$$\text{Market-maker's profit} \approx -\frac{\varepsilon^2}{2}\Gamma_t - h\theta_t - rh(S_t\Delta_t - V_t)$$

Statement I is false, since the change in the stock price, ε, is squared in the formula above.

Statement II is true since time decay reduces the value of the option over time.

Statement III is poorly worded. The first sentence of Statement III is not always true, so it would be reasonable for a student to conclude that Statement III is false.

The ambiguity in Statement III does not prevent us from obtaining the correct answer. Since Statement I is false, the only possible correct answer is D.

Since Statement III is poorly worded, so let's consider a clearer version: "If in order to hedge, the market-maker must purchase stock, then the net carrying cost is a component of the overall cost." This statement is true, because the market-maker must pay interest on any funds that are borrowed.

The exam's version of Statement III appears at the bottom of page 420 in the textbook.

Solution 34

E Chapter 10, Barrier Options

All of the options have the same strike price. We can use the parity relationship for barrier options to find the value of a regular option. Making use of the down-and-out and down-and-in options that have a barrier of $30,000, we have:

$$\text{Knock-in option} + \text{Knock-out option} = \text{Ordinary option}$$

$$30 + 25 = 55$$

Therefore, the value of the ordinary option is $55. We can use the parity relationship again, this time with the up-and-in and up-and-out options that have a barrier of $50,000:

$$\text{Knock-in option} + \text{Knock-out option} = \text{Ordinary option}$$

$$X + 15 = 55$$
$$X = 40$$

The value of the up-and-in option with a barrier of $50,000 is $40.

Solution 35

B Chapter 15, Itô's Lemma

This question contains a typo. The correct answer to the question as worded is:

$$dS(t) = (\alpha - \delta - \tfrac{1}{2}\sigma^2)S(t)dt + \sigma dZ(t)$$

This correct answer does not appear as one of the choices, though.

Based on the fact that Choice B is listed as the correct answer, it seems clear that a parenthesis was misplaced in the first bullet point in the question. The corrected expression is:

$$S(t) = S(0)\exp\left((\alpha - \delta - \tfrac{1}{2}\sigma^2)t + \sigma Z(t)\right)$$

The solution below is based on the corrected expression for $S(t)$ shown above.

A lognormal stock price implies that changes in the stock price follow geometric Brownian motion:

$$S(t) = S(0)e^{(\alpha-\delta-0.5\sigma^2)t+\sigma Z(t)} \quad \Leftrightarrow \quad dS(t) = (\alpha-\delta)S(t)dt + \sigma S(t)dZ(t)$$

The expression in Choice B is equal to $dS(t)$:

$$\begin{aligned} dS(t) &= (\alpha-\delta)S(t)dt + \sigma S(t)dZ(t) \\ &= (\alpha-\delta)S(t)dt + \sigma S(t)dZ(t) + \tfrac{1}{2}\sigma^2 S(t)dt - \tfrac{1}{2}\sigma^2 S(t)dt \\ &= (\alpha-\delta-\tfrac{1}{2}\sigma^2)S(t)dt + \sigma S(t)dZ(t) + \tfrac{1}{2}\sigma^2 S(t)dt \end{aligned}$$

Solution 36

E Chapter 19, Vasicek Model

This question contains a typo. Before we answer it, let's make 3 modifications:

- *In the first line, "Vasieck" should be "Vasicek."*
- *The third bullet point should be:*

  r = 0.05

- *The final line should be, "Calculate the expected change in the interest rate, expressed as an annual rate."*

The process can be written as:

$$dr = 0.15(0.10 - r)dt + 0.05dZ$$

The expected change in the interest rate is:

$$E[dr] = E[0.15(0.10 - r)dt + 0.05dZ] = (0.015 - 0.15r)dt + 0.05 \times 0$$
$$= (0.015 - 0.15r)dt$$

Since $r = 0.05$, we have:

$$E[dr] = (0.015 - 0.15r)dt = \left[0.015 - 0.15(0.05)\right]dt = 0.0075dt$$

To convert the expected change in the interest rate into an annual rate, we must divide by the increment of time:

$$\frac{0.0075dt}{dt} = 0.0075$$

Solution 37

B Chapter 18, Black-Derman-Toy Model

This question contains a typo. Just under the table, the phrase should be, "where volatility refers to the volatility of the bond ***yield*** *one year from today."*

The yield volatility for the 3-year bond is:

$$\text{Yield volatility} = 0.5 \times \ln\left[\frac{P(1,T,r_u)^{-1/(T-1)} - 1}{P(1,T,r_d)^{-1/(T-1)} - 1}\right] = 0.5 \times \ln\left[\frac{P(1,3,r_u)^{-1/2} - 1}{P(1,3,r_d)^{-1/2} - 1}\right]$$
$$= 0.5 \times \ln\left[\frac{0.8133^{-1/2} - 1}{0.8537^{-1/2} - 1}\right] = 0.1398$$