

# CAS/CIA Exam 3

## Fall 2007 Solutions

*Note: The CAS/CIA Spring 2007 Exam 3 contained 17 questions that were relevant for SOA Exam MFE. The solutions to those questions are included in this document.*

### Solution 13

**D** Chapter 2, Comparing Options

American options are always worth at least as much as otherwise equivalent European options:

$$C_{Amer}(S, K, T) \geq C_{Eur}(S, K, T)$$

Therefore, Option B is at least as valuable as Option A, and Option D is at least as valuable Option C:

$$B \geq A$$

$$D \geq C$$

An American option with a longer time to expiration is at least as valuable as an otherwise equivalent American option with a shorter time to expiration:

$$C_{Amer}(S_0, K, T) \geq C_{Amer}(S_0, K, t) \quad \text{where: } t < T$$

Therefore, Option D is at least as valuable as Option B:

$$D \geq B$$

A call with a lower strike price is at least as valuable as an otherwise equivalent call with a higher strike price:

$$C(K_1) \geq C(K_2) \quad \text{where: } K_1 < K_2$$

Therefore, Option D is at least as valuable as Option E.

$$D \geq E$$

### Solution 14

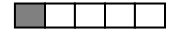
**D** Chapter 1, Put-Call Parity

We can use put-call parity to find the value of the put option:

$$C_{Eur}(K, T) + Ke^{-rT} = S_0 - PV_{0,T}(Div) + P_{Eur}(K, T)$$

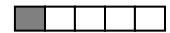
$$5.50 + 47e^{-0.05(2)} = 45 - 1.50e^{-0.05(1)} + P_{Eur}(47, 2)$$

$$P_{Eur}(47, 2) = 4.45$$

**Solution 15****D** Chapter 1, Put-Call Parity

We can use put-call parity to find the risk-free rate on euros:

$$\begin{aligned}
 C_{Eur}(K, T) + Ke^{-rT} &= x_0 e^{-r_f T} + P_{Eur}(K, T) \\
 0.06 + 1.30e^{-0.07(0.75)} &= 1.20e^{r_f(0.75)} + 0.18 \\
 0.9279 &= e^{-r_f(0.75)} \\
 r_f &= 0.0997
 \end{aligned}$$

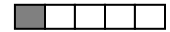
**Solution 16****A** Chapter 1, Put-Call Parity

The net cost to purchase a call and sell a put can be found using put-call parity:

$$\begin{aligned}
 C_{Eur}(K, T) - P_{Eur}(K, T) &= S_0 - Ke^{-rT} \\
 C_{Eur}(K, T) - P_{Eur}(K, T) &= 80 - 82e^{-0.03(180/365)} = -0.7958
 \end{aligned}$$

The net cost to buy 100 calls and sell 100 puts is:

$$100 \times -0.7958 = -79.58$$

**Solution 17****C** Chapter 4, Delta

If the stock price increases to \$120, the value of the option will be:

$$120 - 105 = 15$$

If the stock price decreases to \$90, then the option will be worthless.

The delta for the call option is:

$$\Delta = e^{-\delta h} \frac{V_u - V_d}{S(u - d)} = e^{0(1)} \frac{15 - 0}{120 - 90} = 0.5$$

**Solution 18****E** Chapter 4, Two-Period Binomial Model*Since no dividend is mentioned, we assume that the dividend rate is zero.*

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.05-0.00)(0.5)} - 0.85}{1.25 - 0.85} = 0.43829$$

The stock price tree and its corresponding tree of option prices are:

Stock		American Put	
	112.5000		0.0000
90.0000		1.9175	
72.0000	76.5000	11.1191	3.5000
	61.2000	<b>18.8000</b>	
	52.0200		27.9800

If the stock price falls to \$61.20 at the end of 6 months, the option is exercised early because its exercise value of \$18.80 is greater than the value of holding it:

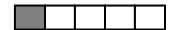
$$V = e^{-0.05(0.5)}[0.43829 \times 3.50 + (1 - 0.43829)27.98] = 16.8248$$

The value of the American put option is:

$$V = e^{-0.05(0.5)}[0.43829 \times 1.9179 + (1 - 0.43829)18.80] = 11.1191$$

### Solution 19

#### B Chapter 3, One-Period Binomial Model



The values of  $u$  and  $d$  are:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.04-0.00)(0.25) + 0.15\sqrt{0.25}} = 1.08872$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.04-0.00)(0.25) - 0.15\sqrt{0.25}} = 0.93707$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.04-0.00)(0.25)} - 0.93707}{1.08872 - 0.93707} = 0.48126$$

The stock price tree and its corresponding tree of option prices are:

Stock		European Call	
	10.8872		0.3872
10.0000		0.1845	
	9.3707		0.0000

The value of the call option is:

$$e^{-0.04(0.25)}[(0.48126)(0.3872) + (1 - 0.48126)(0.00)] = 0.1845$$

**Solution 20****B** Chapter 7, Black-Scholes Option Pricing Model 

Stock returns, not prices, are normally distributed, so A is false.

The standard deviation of the continuously compounded returns,  $\sigma$ , is also known as the stock's volatility or the stock price volatility. Therefore, B is true.

Although the ratio of the stock price to an earlier stock price is lognormally distributed in the Black-Scholes model, changes in the stock prices are not. That is:


$$\ln \frac{S(t)}{S(0)} = \ln S(t) - \ln S(0) \text{ is normally distributed, but}$$

$$\ln[S(t+h) - S(t)] \text{ is not normally distributed}$$

Therefore, C is false.

The Black-Scholes model is based on an assumption that there are no transaction costs. Therefore D is false.

In the Black-Scholes model, the risk-free rate is assumed to be constant. Therefore, E is false.

**Solution 21****E** Chapter 7, Black-Scholes Formula for Currency Options 

The values of  $d_1$  and  $d_2$  are:

$$d_1 = \frac{\ln(x_0/K) + (r - r_f + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(0.82/0.80) + (0.05 - 0.025 + 0.5 \times 0.10^2)(1)}{0.10\sqrt{1}}$$

$$= 0.5469$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.5469 - 0.10\sqrt{1} = 0.4469$$

From the standard normal table, we have:

$$N(d_1) = N(0.5469) \approx N(0.55) = 0.7088$$

$$N(d_2) = N(0.4469) \approx N(0.45) = 0.6736$$

The value of one of the call options is:

$$C_{Eur}(x, K, \sigma, r, T, r_f) = x_0 e^{-r_f T} N(d_1) - K e^{-r T} N(d_2)$$

$$= 0.82 e^{-0.025(1)} (0.7088) - 0.80 e^{-0.05(1)} (0.6736)$$

$$= 0.054267$$

The value of 850 of the options is:

$$850 \times 0.054267 = 46.13$$

**Solution 22****B** Chapter 8, Option ElasticityThe values of  $d_1$  and  $d_2$  are:

$$d_1 = \frac{\ln(S/K) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(25/24) + (0.04 - 0 + 0.5 \times 0.20^2)(1)}{0.20\sqrt{1}} = 0.5041$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.5041 - 0.20\sqrt{1} = 0.3041$$

From the standard normal table, we have:

$$N(d_1) = N(0.5041) \approx N(0.50) = 0.6915$$

$$N(d_2) = N(0.3041) \approx N(0.30) = 0.6179$$

The value of the call option is:

$$\begin{aligned} C_{Eur}(S, K, \sigma, r, T, \delta) &= Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2) \\ &= 25e^{-0(1)}(0.6915) - 24e^{-0.04(1)}(0.6179) = 3.0394 \end{aligned}$$

The delta of the call option is:

$$\Delta_{Call} = e^{-\delta T} N(d_1) = e^{0(1)}(0.6915) = 0.6915$$

The elasticity of the call option is:

$$\Omega = \frac{S\Delta}{V} = \frac{25 \times 0.6915}{3.0394} = 5.69$$

**Solution 23****B** Chapter 3, Single-Period Binomial ModelThe factors for  $u$  and  $d$  are:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.06-0.01)(1) + \sigma\sqrt{1}} = e^{0.05+\sigma}$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.06-0.01)(1) - \sigma\sqrt{1}} = e^{0.05-\sigma}$$

The fact that  $\sigma > 0.05$  implies  $d < 1$ , which in turn implies that if the stock price falls, it will be less than \$10. Therefore, the call option has a payoff of zero if the stock price falls.

The risk-neutral probability that the stock price increases is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{0.05} - e^{0.05-\sigma}}{e^{0.05+\sigma} - e^{0.05-\sigma}} = \frac{1 - e^{-\sigma}}{e^{\sigma} - e^{-\sigma}}$$

We can now solve for  $\sigma$  :

$$\begin{aligned}
 V &= e^{-rh} [(p^*)V_u + (1 - p^*)V_d] \\
 0.9645 &= e^{-0.06} \left[ \frac{1 - e^{-\sigma}}{e^\sigma - e^{-\sigma}} (10e^{0.05+\sigma} - 10) + \left( 1 - \frac{1 - e^{-\sigma}}{e^\sigma - e^{-\sigma}} \right) 0 \right] \\
 &= e^{-0.06} \left[ \frac{e^\sigma}{e^\sigma} \times \frac{1 - e^{-\sigma}}{e^\sigma - e^{-\sigma}} (10.5127e^\sigma - 10) \right] \\
 0.9645 &= e^{-0.06} \frac{e^\sigma - 1}{e^{2\sigma} - 1} (10.5127e^\sigma - 10) \\
 0.9645 &= \frac{e^\sigma - 1}{(e^\sigma + 1)(e^\sigma - 1)} (9.9005e^\sigma - 9.4176) \\
 0.9645(e^\sigma + 1) &= 9.9005e^\sigma - 9.4176 \\
 -8.9360e^\sigma &= -10.3821 \\
 e^\sigma &= 1.1618 \\
 \sigma &= 0.15
 \end{aligned}$$

### Solution 24

**E** Chapter 9, Delta-Gamma-Hedging



The gamma of the stock is zero, so the gamma of the position to be hedged is the gamma of the 100 40-strike puts:

$$100 \times (0.25) = 25$$

We can now solve for  $Q$ , the quantity of the 35-strike put options that must be purchased to bring the hedged portfolio's gamma to zero:

$$25 + 0.5Q = 0$$

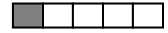
$$Q = -50$$

Since  $Q$  is negative, 50 of the 35-strike put options should be written. The delta of the position becomes:

$$100(-0.05) + 5(1) - 50(-0.10) = 5$$

The quantity of the stock that must be purchased is the opposite of the delta of the position to be hedged, so 5 shares of stock must be sold.

We have found that 50 of the 35-strike put options should be written and 5 shares of stock should be sold to delta-gamma hedge the portfolio.

**Solution 25****B** Chapter 3, One-Period Binomial Model

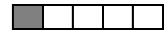
*The question should have stated that the probability provided is a risk-neutral probability.*

If a hurricane does not occur, then the payoff of the policy is zero. If a hurricane occurs, the payoff is the loss minus the deductible:

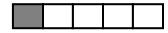
$$1,000,000 - 50,000 = 950,000$$

The value of the payoff is:

$$\begin{aligned} V &= e^{-rh} [(p^*)V_u + (1 - p^*)V_d] \\ &= e^{-0.05(0.25)} [(0.20)(950,000) + (0.80)(0.00)] \\ &= 187,639.78 \end{aligned}$$

**Solution 26****B** Chapter 10, Exotic Options

Barrier options have a lower premium than standard options because barrier options never pay more than standard options, but they might pay less.

**Solution 27****B** Chapter 10, Asian Options

The geometric average price is:

$$[1.27 \times 4.11 \times 5.10 \times 5.50 \times 5.13 \times 4.70]^{1/6} = 3.902$$

The payoff of the geometric average price call is:

$$3.902 - 3.50 = 0.402$$

**Solution 28**

**C** Chapter 10, Compound Options



The risk-neutral probability that the stock price increases in value is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.05-0.00)(0.5)} - 33.20/40}{50.80/40 - 33.20/40} = 0.44390$$

The price trees for the stock and the underlying American put option are below:

Stock		American Put		
		64.52		0.00
	50.80		0.46	
40.00		42.16	5.51	0.84
	33.20		<b>9.80</b>	
		27.56		15.44

*It is not necessary to compute the time 0 price of the American put option (\$5.51) to answer this question, but it is included above for completeness.*

At the end of 6 months, the compound call has value only if the put option is worth more than \$3. This occurs only if the stock price falls to \$33.20 and the American put option's value is \$9.80, in which case the payoff of the compound call is:

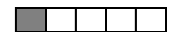
$$9.80 - 3.00 = 6.80$$

The value of the compound call option is:

$$\begin{aligned} V &= e^{-rh} [(p^*)V_u + (1 - p^*)V_d] \\ &= e^{-0.05(0.5)} [(0.44390)(0.00) + (1 - 0.44390)(6.80)] \\ &= 3.688 \end{aligned}$$

**Solution 29**

**B** Chapter 8, Sharpe Ratio



The Sharpe ratio of a call option is the same as the Sharpe ratio of the underlying stock:

$$\text{Sharpe ratio for call option} = \frac{\gamma_{Call} - r}{\sigma_{Call}} = \frac{\alpha - r}{\sigma}$$

The Sharpe ratio for the underlying stock is:

$$\frac{\alpha - r}{\sigma} = \frac{0.08 - 0.04}{0.25} = 0.16$$

Therefore, the Sharpe ratio for the call option is 0.16.