CAS/CIA Exam 3
Fall 2007 Solutions

Note: The CAS/CIA Spring 2007 Exam 3 contained 17 questions that were relevant for SOA Exam MFE. The solutions to those questions are included in this document.

Solution 13

D Chapter 2, Comparing Options

American options are always worth at least as much as otherwise equivalent European options:

$$C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T)$$

Therefore, Option B is at least as valuable as Option A, and Option D is at least as valuable Option C:

$$B \geq A$$
$$D \geq C$$

An American option with a longer time to expiration is at least as valuable as an otherwise equivalent American option with a shorter time to expiration:

$$C_{Amer}(S_0,K,T) \geq C_{Amer}(S_0,K,t) \quad \text{where: } t < T$$

Therefore, Option D is at least as valuable as Option B:

$$D \geq B$$

A call with a lower strike price is at least as valuable as an otherwise equivalent call with a higher strike price:

$$C(K_1) \geq C(K_2) \qquad \text{where: } K_1 < K_2$$

Therefore, Option D is at least as valuable as Option E.

$$D \geq E$$

Solution 14

D Chapter 1, Put-Call Parity

We can use put-call parity to find the value of the put option:

$$C_{Eur}(K,T) + Ke^{-rT} = S_0 - PV_{0,T}(Div) + P_{Eur}(K,T)$$
$$5.50 + 47e^{-0.05(2)} = 45 - 1.50e^{-0.05(1)} + P_{Eur}(47,2)$$
$$P_{Eur}(47,2) = 4.45$$

Solution 15

D Chapter 1, Put-Call Parity

We can use put-call parity to find the risk-free rate on euros:

$$C_{Eur}(K,T)+Ke^{-rT}=x_0e^{-r_fT}+P_{Eur}(K,T)$$
$$0.06+1.30e^{-0.07(0.75)}=1.20e^{r_f(0.75)}+0.18$$
$$0.9279=e^{-r_f(0.75)}$$
$$r_f=0.0997$$

Solution 16

A Chapter 1, Put-Call Parity

The net cost to purchase a call and sell a put can be found using put-call parity:

$$C_{Eur}(K,T)-P_{Eur}(K,T)=S_0-Ke^{-rT}$$
$$C_{Eur}(K,T)-P_{Eur}(K,T)=80-82e^{-0.03(180/365)}=-0.7958$$

The net cost to buy 100 calls and sell 100 puts is:

$$100\times-0.7958=-79.58$$

Solution 17

C Chapter 4, Delta

If the stock price increases to $120, the value of the option will be:

$$120-105=15$$

If the stock price decreases to $90, then the option will be worthless.

The delta for the call option is:

$$\Delta=e^{-\delta h}\frac{V_u-V_d}{S(u-d)}=e^{0(1)}\frac{15-0}{120-90}=0.5$$

Solution 18

E Chapter 4, Two-Period Binomial Model

Since no dividend is mentioned, we assume that the dividend rate is zero.

The risk-neutral probability of an upward movement is:

$$p^*=\frac{e^{(r-\delta)h}-d}{u-d}=\frac{e^{(0.05-0.00)(0.5)}-0.85}{1.25-0.85}=0.43829$$

The stock price tree and its corresponding tree of option prices are:

Stock			American Put		
		112.5000			0.0000
	90.0000			1.9175	
72.0000		76.5000	11.1191		3.5000
	61.2000			**18.8000**	
		52.0200			27.9800

If the stock price falls to \$61.20 at the end of 6 months, the option is exercised early because its exercise value of \$18.80 is greater than the value of holding it:

$$V = e^{-0.05(0.5)}[0.43829 \times 3.50 + (1 - 0.43829)27.98] = 16.8248$$

The value of the American put option is:

$$V = e^{-0.05(0.5)}[0.43829 \times 1.9179 + (1 - 0.43829)18.80] = 11.1191$$

Solution 19

B Chapter 3, One-Period Binomial Model

The values of u and d are:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.04-0.00)(0.25)+0.15\sqrt{0.25}} = 1.08872$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.04-0.00)(0.25)-0.15\sqrt{0.25}} = 0.93707$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.04-0.00)(0.25)} - 0.93707}{1.08872 - 0.93707} = 0.48126$$

The stock price tree and its corresponding tree of option prices are:

Stock		European Call	
	10.8872		0.3872
10.0000		0.1845	
	9.3707		0.0000

The value of the call option is:

$$e^{-0.04(0.25)}[(0.48126)(0.3872) + (1 - 0.48126)(0.00)] = 0.1845$$

Solution 20

B Chapter 7, Black-Scholes Option Pricing Model

Stock returns, not prices, are normally distributed, so A is false.

The standard deviation of the continuously compounded returns, σ, is also known as the stock's volatility or the stock price volatility. Therefore, B is true.

Although the ratio of the stock price to an earlier stock price is lognormally distributed in the Black-Scholes model, changes in the stock prices are not. That is:

$$\ln \frac{S(t)}{S(0)} = \ln S(t) - \ln S(0) \text{ is normally distributed, but}$$

$$\ln\left[S(t+h) - S(t)\right] \text{ is not normally distributed}$$

Therefore, C is false.

The Black-Scholes model is based on an assumption that there are no transaction costs. Therefore D is false.

In the Black-Scholes model, the risk-free rate is assumed to be constant. Therefore, E is false.

Solution 21

E Chapter 7, Black-Scholes Formula for Currency Options

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln(x_0 / K) + (r - r_f + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(0.82/0.80) + (0.05 - 0.025 + 0.5 \times 0.10^2)(1)}{0.10\sqrt{1}}$$
$$= 0.5469$$
$$d_2 = d_1 - \sigma\sqrt{T} = 0.5469 - 0.10\sqrt{1} = 0.4469$$

From the standard normal table, we have:

$$N(d_1) = N(0.5469) \approx N(0.55) = 0.7088$$
$$N(d_2) = N(0.4469) \approx N(0.45) = 0.6736$$

The value of one of the call options is:

$$C_{Eur}(x, K, \sigma, r, T, r_f) = x_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$
$$= 0.82e^{-0.025(1)}(0.7088) - 0.80e^{-0.05(1)}(0.6736)$$
$$= 0.054267$$

The value of 850 of the options is:

$$850 \times 0.054267 = 46.13$$

Solution 22

B Chapter 8, Option Elasticity

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln(S/K) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(25/24) + (0.04 - 0 + 0.5 \times 0.20^2)(1)}{0.20\sqrt{1}} = 0.5041$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.5041 - 0.20\sqrt{1} = 0.3041$$

From the standard normal table, we have:

$$N(d_1) = N(0.5041) \approx N(0.50) = 0.6915$$
$$N(d_2) = N(0.3041) \approx N(0.30) = 0.6179$$

The value of the call option is:

$$C_{Eur}(S,K,\sigma,r,T,\delta) = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$$
$$= 25e^{-0(1)}(0.6915) - 24e^{-0.04(1)}(0.6179) = 3.0394$$

The delta of the call option is:

$$\Delta_{Call} = e^{-\delta T}N(d_1) = e^{0(1)}(0.6915) = 0.6915$$

The elasticity of the call option is:

$$\Omega = \frac{S\Delta}{V} = \frac{25 \times 0.6915}{3.0394} = 5.69$$

Solution 23

B Chapter 3, Single-Period Binomial Model

The factors for u and d are:

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{(0.06-0.01)(1) + \sigma\sqrt{1}} = e^{0.05+\sigma}$$
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{(0.06-0.01)(1) - \sigma\sqrt{1}} = e^{0.05-\sigma}$$

The fact that $\sigma > 0.05$ implies $d < 1$, which in turn implies that if the stock price falls, it will be less than \$10. Therefore, the call option has a payoff of zero if the stock price falls.

The risk-neutral probability that the stock price increases is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{0.05} - e^{0.05-\sigma}}{e^{0.05+\sigma} - e^{0.05-\sigma}} = \frac{1 - e^{-\sigma}}{e^{\sigma} - e^{-\sigma}}$$

We can now solve for σ :

$$V = e^{-rh}\left[(p^*)V_u + (1-p^*)V_d\right]$$

$$0.9645 = e^{-0.06}\left[\frac{1-e^{-\sigma}}{e^{\sigma}-e^{-\sigma}}\left(10e^{0.05+\sigma}-10\right)+\left(1-\frac{1-e^{-\sigma}}{e^{\sigma}-e^{-\sigma}}\right)0\right]$$

$$= e^{-0.06}\left[\frac{e^{\sigma}}{e^{\sigma}}\times\frac{1-e^{-\sigma}}{e^{\sigma}-e^{-\sigma}}\left(10.5127e^{\sigma}-10\right)\right]$$

$$0.9645 = e^{-0.06}\,\frac{e^{\sigma}-1}{e^{2\sigma}-1}\left(10.5127e^{\sigma}-10\right)$$

$$0.9645 = \frac{e^{\sigma}-1}{(e^{\sigma}+1)(e^{\sigma}-1)}\left(9.9005e^{\sigma}-9.4176\right)$$

$$0.9645(e^{\sigma}+1) = 9.9005e^{\sigma}-9.4176$$

$$-8.9360e^{\sigma} = -10.3821$$

$$e^{\sigma} = 1.1618$$

$$\sigma = 0.15$$

Solution 24

E Chapter 9, Delta-Gamma-Hedging

The gamma of the stock is zero, so the gamma of the position to be hedged is the gamma of the 100 40-strike puts:

$$100\times(0.25) = 25$$

We can now solve for Q, the quantity of the 35-strike put options that must be purchased to bring the hedged portfolio's gamma to zero:

$$25 + 0.5Q = 0$$

$$Q = -50$$

Since Q is negative, 50 of the 35-strike put options should be written. The delta of the position becomes:

$$100(-0.05) + 5(1) - 50(-0.10) = 5$$

The quantity of the stock that must be purchased is the opposite of the delta of the position to be hedged, so 5 shares of stock must be sold.

We have found that 50 of the 35-strike put options should be written and 5 shares of stock should be sold to delta-gamma hedge the portfolio.

Solution 25

B Chapter 3, One-Period Binomial Model

The question should have stated that the probability provided is a risk-neutral probability.

If a hurricane does not occur, then the payoff of the policy is zero. If a hurricane occurs, the payoff is the loss minus the deductible:

$$1{,}000{,}000 - 50{,}000 = 950{,}000$$

The value of the payoff is:

$$\begin{aligned} V &= e^{-rh}\left[(p^*)V_u + (1-p^*)V_d\right] \\ &= e^{-0.05(0.25)}\left[(0.20)(950{,}000) + (0.80)(0.00)\right] \\ &= 187{,}639.78 \end{aligned}$$

Solution 26

B Chapter 10, Exotic Options

Barrier options have a lower premium than standard options because barrier options never pay more than standard options, but they might pay less.

Solution 27

B Chapter 10, Asian Options

The geometric average prices is:

$$\left[1.27 \times 4.11 \times 5.10 \times 5.50 \times 5.13 \times 4.70\right]^{1/6} = 3.902$$

The payoff of the geometric average price call is:

$$3.902 - 3.50 = 0.402$$

Solution 28

C Chapter 10, Compound Options

The risk-neutral probability that the stock price increases in value is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.05-0.00)(0.5)} - 33.20/40}{50.80/40 - 33.20/40} = 0.44390$$

The price trees for the stock and the underlying American put option are below:

Stock			American Put		
		64.52			0.00
	50.80			0.46	
40.00		42.16	5.51		0.84
	33.20			**9.80**	
		27.56			15.44

It is not necessary to compute the time 0 price of the American put option ($5.51) to answer this question, but it is included above for completeness.

At the end of 6 months, the compound call has value only if the put option is worth more than $3. This occurs only if the stock price falls to $33.20 and the American put option's value is $9.80, in which case the payoff of the compound call is:

$$9.80 - 3.00 = 6.80$$

The value of the compound call option is:

$$\begin{aligned} V &= e^{-rh}\left[(p^*)V_u + (1-p^*)V_d\right] \\ &= e^{-0.05(0.5)}\left[(0.44390)(0.00) + (1-0.44390)(6.80)\right] \\ &= 3.688 \end{aligned}$$

Solution 29

B Chapter 8, Sharpe Ratio

The Sharpe ratio of a call option is the same as the Sharpe ratio of the underlying stock:

$$\text{Sharpe ratio for call option} = \frac{\gamma_{Call} - r}{\sigma_{Call}} = \frac{\alpha - r}{\sigma}$$

The Sharpe ratio for the underlying stock is:

$$\frac{\alpha - r}{\sigma} = \frac{0.08 - 0.04}{0.25} = 0.16$$

Therefore, the Sharpe ratio for the call option is 0.16.

