Course MFE/3F Practice Exam 1 – Solutions

Solution 1

C Chapter 21, Sharpe Ratio

If we (incorrectly) assume that the <u>cost</u> of the shares needed to delta-hedge the put option is $18.25, then we have:

$S\Delta = 18.25 \quad \Rightarrow \quad \Delta$ *is positive*

But the delta of a put option must be negative, so the equation above cannot be correct. Since (iv) tells us that the put option is delta-hedged by <u>selling</u> shares of stock, the original position must be a short position in the put option.

The cost of the shares required to delta-hedge the put option is the number of shares required, Δ, times the cost of each share, S. Since shares are sold, the cost is negative. Therefore, based on statement (iv) in the question, we have:

$$\Delta \times S = -18.25$$

The elasticity of the put option is:

$$\Omega = \frac{S\Delta}{V} = \frac{-18.25}{7}$$

The risk premium of the put option is equal to the elasticity of the put option times the risk premium of the underlying bond:

$$\gamma - r = \Omega \times (\alpha - r)$$

$$\gamma - 0.08 = \frac{-18.25}{7} \times (0.11 - 0.08)$$

$$\gamma = 0.0018$$

Solution 2

D Chapter 24, Black Formula

The option expires in 1 year, so $T = 1$. The underlying bond matures 1 year after the option expires, so $s = 1$. The bond forward price is:

$$F = P_0(T, T+s) = \frac{P(0, T+s)}{P(0,T)} = \frac{0.842}{0.926} = 0.90929$$

The volatility of the forward price is:

$$\sigma = 0.08$$

We have:

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + 0.5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{0.90929}{0.93}\right) + 0.5(0.08)^2(1)}{0.08\sqrt{1}} = -0.24154$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.24154 - 0.08\sqrt{1} = -0.32154$$

$$N(-d_1) = N(0.24154) = 0.59543$$

$$N(-d_2) = N(0.32154) = 0.62610$$

The Black formula for the put price is:

$$\begin{aligned} P &= P(0,T)\left[K \times N(-d_2) - F \times N(-d_1)\right] \\ &= 0.926\left[0.930 \times 0.62610 - 0.90929 \times 0.59543\right] \\ &= 0.03783 \end{aligned}$$

Solution 3

C Chapter 10, Two-Period Binomial Model

The factors u and d are constant in the model since:

$$u = \frac{120}{100} = \frac{144}{120} = \frac{96}{80} = 1.20$$

$$d = \frac{80}{100} = \frac{96}{120} = \frac{64}{80} = 0.80$$

The risk-neutral probability is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.05-0.00)1} - 0.8}{1.20 - 0.80} = 0.62818$$

If the final stock price is \$144, then the payoff of the call option is \$50. If the final stock price is \$96, then the payoff of the call option is \$2. If the final stock price is \$64, then the payoff of the call option is \$0. The value of the call option is:

$$\begin{aligned} V(S_0, K, 0) &= e^{-r(hn)} \sum_{j=0}^{n} \left[\binom{n}{j} (p^*)^j (1-p^*)^{n-j} V(S_0 u^j d^{n-j}, K, hn)\right] \\ &= e^{-0.05(2)}[(0.62818)^2(50) + 2(0.62818)(1-0.62818)2 + 0] \\ &= 18.698 \end{aligned}$$

Solution 4

C Chapter 12, Elasticity

The textbook discusses all three of the statements in the last two paragraphs on page 391.

Statement I is false. The elasticity of a call option decreases as the option becomes more in the money. As the strike price decreases, the option becomes more in the money, and therefore the elasticity decreases.

Statement II is false. The upper bound for a put option is 0, not −1. The correct statement is $\Omega_{Put} \le 0$.

Statement III is true. The elasticity is equal to the percentage of the replicating portfolio invested in the stock. Since a call option is replicated by a leveraged investment in the stock, $\Omega_{Call} \ge 1$.

Solution 5

E Chapter 22, All-or-Nothing Options

For the square of the final stock price to be greater than 64, the final stock price must be greater than 8:

$$\left[S(1)\right]^2 > 64 \qquad \Leftrightarrow \qquad S(1) > 8$$

Therefore, the option described in the question is 512 cash-or-nothing call options that have a strike price of 8. The current value of the option is:

$$512 \times \text{CashCall}(S,K,T) = 512 \times e^{-rT} N(d_2) = 512 \times e^{-0.03} N(d_2)$$

To find the delta of the option, we must find the derivative of the price with respect to the stock price:

$$\frac{\partial\left(512 \times e^{-0.03} N(d_2)\right)}{\partial S} = 512 e^{-0.03} \frac{\partial\left(N(d_2)\right)}{\partial S} = 512 e^{-0.03} N'(d_2) \frac{\partial d_2}{\partial S}$$

The derivative of d_2 with respect to the stock price is:

$$\frac{\partial d_2}{\partial S} = \frac{\partial\left(d_1 - \sigma\sqrt{T}\right)}{\partial S} = \frac{\partial\left(\dfrac{\ln\left(\dfrac{Se^{-\delta T}}{Ke^{-rT}}\right) + \dfrac{\sigma^2}{2}T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right)}{\partial S} = \frac{\dfrac{Ke^{-rT}}{Se^{-\delta T}} \times \dfrac{e^{-\delta T}}{Ke^{-rT}}}{\sigma\sqrt{T}} = \frac{1}{S\sigma\sqrt{T}}$$

$$= \frac{1}{8 \times 0.30 \times 1} = 0.41667$$

The current value of d_2 is:

$$d_2 = \frac{\ln\left(\frac{Se^{-\delta T}}{Ke^{-rT}}\right) - \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{8e^{-0.02}}{8e^{-0.03}}\right) - \frac{0.30^2}{2}}{0.30} = -0.11667$$

The density function for the standard normal random variable is:

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$$

We can now calculate the delta of the option:

$$\begin{aligned} 512e^{-0.03} N'(d_2) \frac{\partial d_2}{\partial S} &= 512e^{-0.03} \frac{1}{\sqrt{2\pi}} e^{-0.5d_2^2} \times 0.41667 \\ &= 512e^{-0.03} \frac{1}{\sqrt{2\pi}} \times e^{-0.5(-0.11667)^2} \times 0.41667 \\ &= 512 \times 0.16022 = 82.0322 \end{aligned}$$

Therefore, 82.0322 shares must be purchased to delta-hedge the option.

Solution 6

C Chapter 20, Valuing a Claim on S^a

The stock price follows geometric Brownian motion, and the claim pays $S(T)^a$, where $a = 5$. The forward price on the claim is therefore:

$$\begin{aligned} F_{0,T}\left[S(T)^a\right] &= S(0)^a e^{(r-\delta^*)(T-0)} \\ &= S(0)^a e^{\left[a(r-\delta)+0.5a(a-1)\sigma^2\right]T} \\ &= \left[S(0)e^{(r-\delta)T}\right]^a e^{0.5a(a-1)\sigma^2 T} \\ &= \left[F_{0,T}(S)\right]^a e^{0.5a(a-1)\sigma^2 T} \\ &= (1.25)^5 e^{0.5(5)(5-1)(0.25)^2(2)} \\ &= 10.65 \end{aligned}$$

Solution 7

C Chapter 13, Delta-Gamma Hedging

The gamma of the position to be hedged is:

$$-1{,}000 \times 0.0422 = -42.20$$

We can solve for the quantity, Q, of the other put option that must be purchased to bring the hedged portfolio's gamma to zero:

$$-42.2 + 0.0282Q = 0.00$$
$$Q = 1{,}496.5$$

The delta of the position becomes:

$$-1{,}000 \times (-0.5875) + 1{,}496.5 \times (-0.3392) = 79.9$$

The quantity of underlying stock that must be purchased, Q_S, is the opposite of the delta of the position being hedged:

$$Q_S = -79.9$$

Therefore, in order to delta-hedge and gamma-hedge the position, we must sell 79.9 units of stock and purchase 1,496.5 units of Put-II.

Solution 8

B Chapter 20, Geometric Brownian Motion Equivalencies

A lognormal stock price implies that changes in the stock price follow geometric Brownian motion:

$$S(t) = S(0)e^{(\alpha-\delta-0.5\sigma^2)t+\sigma Z(t)} \quad \Leftrightarrow \quad dS(t) = (\alpha-\delta)S(t)dt + \sigma S(t)dZ(t)$$

Substituting 0.4 for σ, we have:

$$S(t) = S(0)e^{(\alpha-\delta-0.50\times 0.4^2)t+0.4Z(t)} \quad \Leftrightarrow \quad dS(t) = (\alpha-\delta)S(t)dt + 0.4S(t)dZ(t)$$
$$S(t) = S(0)e^{(\alpha-\delta-0.08)t+0.4Z(t)} \quad \Leftrightarrow \quad dS(t) = (\alpha-\delta)S(t)dt + 0.4S(t)dZ(t)$$

The expression for $dS(t)$ can be rewritten to match Choice B:

$$\begin{aligned} dS(t) &= (\alpha-\delta)S(t)dt + 0.4S(t)dZ(t) \\ &= (\alpha-\delta-0.08)S(t)dt + 0.4S(t)dZ(t) + 0.08S(t)dt \end{aligned}$$

Solution 9

E Chapter 12, Black-Scholes Call Price

The first step is to calculate d_1 and d_2:

$$\begin{aligned} d_1 &= \frac{\ln(S/K) + (r-\delta+0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(30/35) + (0.06-0.02+0.5\times 0.32^2)\times 0.25}{0.32\sqrt{0.25}} \\ &= -0.82094 \\ d_2 &= d_1 - \sigma\sqrt{T} = -0.82094 - 0.32\sqrt{0.25} = -0.98094 \end{aligned}$$

We have:

$$N(d_1) = N(-0.82094) = 0.20584$$

$$N(d_2) = N(-0.98094) = 0.16331$$

The value of one European call option is:

$$C_{Eur} = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$
$$= 30e^{-0.02(0.25)} \times 0.20584 - 35e^{-0.06(0.25)} \times 0.16331 = 0.513649$$

The value of 100 of the European call options is:

$$100 \times 0.513649 = 51.36$$

Solution 10

D Chapter 18, Probability of Future Stock Price

The stock price follows geometric Brownian motion with:

$$\alpha = 0.15 - 0.03 = 0.12$$
$$\sigma = -0.30$$

To answer this question, we use:

$$\text{Prob}\left(S_T < K\right) = N(-\hat{d}_2)$$

When calculating $\hat{d}_2$, we use the absolute value of σ, so below we have $\sigma = 0.3$:

$$\hat{d}_2 = \frac{\ln\left(\frac{S_t}{K}\right) + (\alpha - \delta - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln\left(\frac{S_t}{1.10S_t}\right) + \left(0.15 - 0.03 - 0.5(0.3)^2\right)0.5}{0.3\sqrt{0.5}}$$
$$= -0.27252$$

The probability that the stock price does not increase by more than 10% over the next 6 months is:

$$\text{Prob}\left(S_{t+0.5} < 1.0S_t\right) = N(-\hat{d}_2) = N(0.27252) = 0.60739$$

Solution 11

E Chapter 23, Historical Volatility

The Derivatives Markets textbook provides two formulas for estimating the volatility of the stock:

1. Chapter 11 Formula:

$$\hat{\sigma} = \frac{1}{\sqrt{h}} \times \sqrt{\frac{\sum_{i=1}^{k} (r_i - \overline{r})^2}{k-1}}$$

2. Chapter 23 Formula:

$$\hat{\sigma}_H = \frac{1}{\sqrt{h}} \times \sqrt{\frac{\sum_{i=1}^{k} (r_i)^2}{k-1}}$$

If we were to use the Chapter 11 formula, then we would need to obtain $\overline{r}$:

$$\overline{r} = \frac{\sum_{i=1}^{k} r_i}{k} = \frac{\sum_{i=1}^{14} r_i}{14}$$

Since we are not given the first 9 prices, we are not able to obtain $\overline{r}$. Therefore we use the Chapter 23 formula instead.

The original volatility estimate is based on 10 prices. There are 9 returns, so $k = 9$:

$$\hat{\sigma}_H = \frac{1}{\sqrt{h}} \times \sqrt{\frac{\sum_{i=1}^{k} (r_i)^2}{k-1}}$$

$$0.18 = \frac{1}{\sqrt{1/252}} \times \sqrt{\frac{\sum_{i=1}^{9} (r_i)^2}{9-1}}$$

$$\sum_{i=1}^{9} (r_i)^2 = 0.00102857$$

After a week passes, the analyst can add 5 more returns into the sum of squared returns:

$$\sum_{i=1}^{14}(r_i)^2 = \sum_{i=1}^{9}(r_i)^2 + \sum_{i=10}^{14}(r_i)^2 = 0.00102857 + \sum_{i=10}^{14}(r_i)^2$$

$$= 0.00102857 + \left[\ln\left(\frac{98}{100}\right)\right]^2 + \left[\ln\left(\frac{96}{98}\right)\right]^2 + \left[\ln\left(\frac{92}{96}\right)\right]^2 + \left[\ln\left(\frac{90}{92}\right)\right]^2 + \left[\ln\left(\frac{94}{90}\right)\right]^2$$

$$= 0.00102857 + 0.00501865$$

$$= 0.00604722$$

Now we can calculate the volatility using all 15 prices. There are 14 returns, so $k = 14$:

$$\hat{\sigma}_H = \frac{1}{\sqrt{h}} \times \sqrt{\frac{\sum_{i=1}^{k}(r_i)^2}{k-1}} = \frac{1}{\sqrt{1/252}} \times \sqrt{\frac{\sum_{i=1}^{14}(r_i)^2}{14-1}} = \frac{1}{\sqrt{1/252}} \times \sqrt{\frac{0.00604722}{14-1}} = 0.34.24\%$$

We can use the TI-30XS Multiview calculator to obtain the sum of the 5 additional squared returns above:

[data] [data] 4 (to clear the data table)

(enter the data below)

L1	L2	L3
100	98	-----------
98	96	
96	92	
92	90	
90	94	

(place cursor in the L3 column)

[data] → (to highlight FORMULA)

1 [ln] [data] 2 / [data] 1) [enter]

[2nd] [quit] [2nd] [stat] 1

DATA: (highlight L3) FRQ: (highlight one) (select CALC) [enter]

Statistic number 6 is 0.00501865.

Solution 12

D Chapter 24, Black-Derman-Toy Model

In each column of rates, each rate is greater than the rate below it by a factor of:

$$e^{2\sigma_i\sqrt{h}}$$

Therefore, the missing rate in the third column is:

$$0.2086e^{-2\sigma_i\sqrt{1}} = 0.2086 \times \frac{0.2086}{0.2785} = 0.1562$$

The missing rate in the fourth column is:

$$0.4179e^{-2\sigma_i\sqrt{1}} = 0.4179 \times \frac{0.1473}{0.2085} = 0.2952$$

The tree of short-term rates is then:

			41.79%
		27.85%	
	17.47%		29.52%
14.00%		20.86%	
	14.59%		20.85%
		15.62%	
			14.73%

The caplet pays off only if the interest rate at the end of the fourth year is greater than 24.00%. The payoff table is:

			12.5467
		0.0000	
	0.0000		4.2619
0.0000		0.0000	
	0.0000		0.0000
		0.0000	
			0.0000

The payments have been converted to their equivalents payable at the end of 3 years. The calculations are shown below:

$$\frac{100 \times (0.4179 - 0.2400)}{1.4179} = 12.5467$$

$$\frac{100 \times (0.2952 - 0.2400)}{1.2952} = 4.2645$$

The expected present value of these payments is the value of the 3-year caplet:

$$\begin{aligned} V_0 &= E^*\left[V_T \times \prod_{i=0}^{T-1} \frac{1}{(1+r_i)}\right] \\ &= 0.5^3\left(\frac{12.5467}{(1.14)(1.1747)(1.2785)} + \frac{4.2645}{(1.14)(1.1747)(1.2785)} \right. \\ &\qquad \left. + \frac{4.2645}{(1.14)(1.1747)(1.2086)} + \frac{4.2645}{(1.14)(1.1459)(1.2086)}\right) \\ &= 1.89 \end{aligned}$$

Alternatively, the value of the caplet can be found recursively, as shown in the tree below:

			12.5467
		6.5715	
	3.5466		4.2645
1.8925		1.7609	
	0.7684		0.0000
		0.0000	
			0.0000

Solution 13

D Chapter 9, Currency Options

Let's use the British pound as the base currency. The current exchange rate is:

$$x_0 = 0.38 \text{ pounds}$$

The pound-denominated put is related to the dollar-denominated call. In the equations below, we use \$ to denote Australian dollars:

$$P_{£}(x_0, K, T) = x_0 K C_{\$}\left(\frac{1}{x_0}, \frac{1}{K}, T\right)$$

$$0.03 = (0.38)(0.4) C_{\$}\left(\frac{1}{0.38}, 2.5, 0.5\right)$$

$$C_{\$}\left(\frac{1}{0.38}, 2.5, 0.5\right) = 0.1974$$

<u>Alternative Solution</u>

We can obtain the same answer by thinking carefully about the rights conveyed to the owners of the two options.

The first option gives its owner the right to:

Give up \$1.00 Get 0.4 pounds

The value of this option is:

$$0.03 \text{ pounds} = \frac{0.03}{0.38} \text{ dollars} = \$0.07895$$

The second option gives its owner the right to:

Give up \$2.50 Get 1.0 pound

The payoff is 2.5 times the payoff of the first option, so the value of the option is 2.5 times the value of the first option:

$$2.5 \times \$0.07895 = \$0.1974$$

Solution 14

D Chapter 10, Options on Futures Contracts

The values of u_F and d_F are:

$$u_F = e^{\sigma\sqrt{h}} = e^{0.25\sqrt{0.5}} = 1.19336$$
$$d_F = e^{-\sigma\sqrt{h}} = e^{-0.25\sqrt{0.5}} = 0.83797$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{1-d_F}{u_F - d_F} = \frac{1-0.83797}{1.19336-0.83797} = 0.45592$$

The futures price tree and the call option tree are below:

F_0	$F_{\frac{1}{2}}$	F_1
		1,566.5309
	1,312.7010	
1,100.0000		1,100.0000
	921.7636	
		772.4074

American Call		
		566.5309
	312.7010	
155.3765		100.0000
	42.7228	
		0.0000

If the futures price reaches $1,312.7010, then it is optimal to exercise the call option early since the option exercise value exceeds the expected present value of the option at that node.

The number of futures contracts that the investor must be long is:

$$\Delta = \frac{V_u - V_d}{F(u_F - d_F)} = \frac{312.7010 - 42.7228}{1{,}312.7010 - 921.7636} = 0.6906$$

Solution 15

B Chapter 11, Realistic Probability

In the Cox-Ross-Rubinstein model, the values of u and d are:

$$u = e^{\sigma\sqrt{h}} = e^{0.3\sqrt{0.5}} = 1.23631$$
$$d = e^{-\sigma\sqrt{h}} = e^{-0.3\sqrt{0.5}} = 0.80886$$

We can solve for p, the true probability of the stock price going up, using the following formula:

$$p = \frac{e^{(\alpha-\delta)h} - d}{u-d} = \frac{e^{(0.15-0.05)0.5} - 0.80886}{1.23631-0.80886} = 0.5671$$

Solution 16

A Chapter 12, Options on Futures

The 6-month futures price is:

$$F_{0,T_F} = S_t e^{(r-\delta)(T_F - 0)}$$
$$F_{0,0.5} = 50e^{(0.09-0.05)(0.5)} = 51.0101$$

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln\left(F_{0,T_F}/K\right) + 0.5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln\left(51.0101/55\right) + 0.5(0.30)^2(0.5)}{0.30\sqrt{0.5}} = -0.24895$$
$$d_2 = d_1 - \sigma\sqrt{T} = -0.24895 - 0.30\sqrt{0.5} = -0.46108$$

We have:

$$N(d_1) = N(-0.24895) = 0.40170$$

$$N(d_2) = N(-0.46108) = 0.32237$$

The value of the call option is:

$$\begin{aligned} C_{Eur}(F_{0,T_F}, K, \sigma, r, T, r) &= F_{0,T_F} e^{-rT} N(d_1) - Ke^{-rT} N(d_2) \\ &= 51.0101e^{-0.09(0.5)} \times 0.40170 - 55e^{-0.09(0.5)} \times 0.32237 \\ &= 2.6389 \end{aligned}$$

Solution 17

A Chapter 13, Frequency of Re-Hedging

The gamma the 100 calls is:

$$100 \times 0.0342 = 3.42$$

The return on the delta-hedged position is:

$$R_{h,i} = \frac{1}{2} S^2 \sigma^2 \Gamma (1 - x_i^2) h$$

We can solve for the random values that produce a profit greater than \$1.40:

$$1.40 < \frac{1}{2} \times 75^2 \times 0.30^2 \times 3.42 \times (1 - x_i^2) \times \frac{1}{365}$$
$$1.40 < 2.37175(1 - x_i^2)$$
$$0.59028 < 1 - x_i^2$$
$$x_i^2 < 0.40972$$
$$x_i < \sqrt{0.40972} \qquad \text{and} \qquad x_i > -\sqrt{0.40972}$$
$$x_i < 0.64009 \qquad \text{and} \qquad x_i > -0.64009$$

Since x_i is a standard normal random variable, the probability that both of the inequalities above are satisfied is:

$$N(0.64009) - N(-0.64009) = N(0.64009) - \left[1 - N(0.64009)\right]$$

We have:

$$N(0.64009) = 0.73894$$

The probability of the profit exceeding \$1.40 is therefore:

$$N(0.64009) - \left[1 - N(0.64009)\right] = 0.73894 - (1 - 0.73894) = 0.47788$$

Solution 18

A Chapter 14, Exchange Options

Since we need to find the value of the option in yen, we use yen as the base currency.

Let's define the underlying asset to be 1 euro and the strike asset to be 1.45 Canadian dollars. This makes the option an exchange put option with:

$$S = 165$$
$$K = 1.45 \times 115 = 166.75$$

The volatility of $\ln(S/K)$ is:

$$\sigma = \sqrt{\sigma_S^2 + \sigma_K^2 - 2\rho\sigma_S\sigma_K} = \sqrt{0.18^2 + 0.23^2 - 2(0.59)(0.18)(0.23)} = \sqrt{0.036448}$$
$$= 0.19091$$

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln\left(\dfrac{Se^{-\delta_S T}}{Ke^{-\delta_K T}}\right) + \dfrac{\sigma^2 T}{2}}{\sigma\sqrt{T}} = \frac{\ln\left(\dfrac{165e^{-0.05(1)}}{166.75e^{-0.07(1)}}\right) + \dfrac{0.19091^2(1)}{2}}{0.19091\sqrt{1}} = 0.14495$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.14495 - 0.19091\sqrt{1} = -0.04596$$

We have:

$$N(-d_1) = N(-0.14495) = 0.44238$$

$$N(-d_2) = N(0.04596) = 0.51833$$

The value of the exchange put is:

$$\begin{aligned}\text{ExchangePutPrice} &= Ke^{-\delta_K T}N(-d_2) - Se^{-\delta_S T}N(-d_1) \\ &= 166.75e^{-0.07(1)}(0.51833) - 165e^{-0.05(1)}(0.44238) \\ &= 11.155\end{aligned}$$

Solution 19

E Chapter 9, Put-Call Parity

The dividends paid before the expiration of the options occur at time 1 month, 4 months, and 7 months.

We can use put-call parity to find the value of the put option:

$$C_{Eur}(K,T) + Ke^{-rT} = S_0 - PV_{0,T}(Div) + P_{Eur}(K,T)$$

$$6.38 + 75e^{-0.12(0.75)} = 73 - 2\left(e^{-0.12(1/12)} + e^{-0.12(4/12)} + e^{-0.12(7/12)}\right) + P_{Eur}(75, 0.75)$$

$$P_{Eur}(75, 0.75) = 7.6913$$

Solution 20

D Chapter 24, Risk-Neutral Cox-Ingersoll-Ross Model

Ann uses the CIR model for the short rate. We can use the Sharpe ratio in Ann's model to solve for the parameter $\bar{\phi}$:

$$\phi(0.16, 0) = 1.20$$

$$\bar{\phi}\frac{\sqrt{r}}{\sigma} = 1.20$$

$$\bar{\phi}\frac{\sqrt{0.16}}{0.05} = 1.20$$

$$\bar{\phi} = 0.15$$

The risk-neutral version of Ann's model is:

$$\begin{aligned}
dr &= \left[a(r) + \sigma(r)\phi(r,t)\right]dt + \sigma(r)d\tilde{Z} \\
&= \left[a(r) + \sigma\sqrt{r}\bar{\phi}\frac{\sqrt{r}}{\sigma}\right]dt + \sigma(r)d\tilde{Z} \\
&= \left[a(r) + \bar{\phi}r\right]dt + \sigma(r)d\tilde{Z} \\
&= \left[0.4(0.09375 - r) + 0.15r\right]dt + 0.05\sqrt{r}d\tilde{Z} \\
&= 0.25(0.15 - r)dt + 0.05\sqrt{r}d\tilde{Z}
\end{aligned}$$

Mike's model will produce the same prices (and therefore the same yields) as Ann's model if the risk-neutral version of his model is the same as the risk-neutral version of Ann's model.

We can rule out Choices A and B, because they are based on the Vasicek model. Ann's model is a CIR model.

We can use the Sharpe ratio in Choice C to solve for the parameter $\bar{\phi}$:

$$\begin{aligned}
\bar{\phi}\frac{\sqrt{0.16}}{0.05} &= 0.05 \\
\bar{\phi} &= 0.00625
\end{aligned}$$

The risk-neutral version of Choice C is:

$$\begin{aligned}
dr &= \left[a(r) + \bar{\phi}r\right]dt + \sigma(r)d\tilde{Z} \\
&= \left[0.25(0.15 - r) + 0.00625r\right]dt + 0.05\sqrt{r}d\tilde{Z} \\
&= 0.24375(0.153846 - r)dt + 0.05\sqrt{r}d\tilde{Z}
\end{aligned}$$

The risk-neutral version of Choice C is not the same as the risk-neutral version of Ann's model.

We can use the Sharpe ratio in Choice D to solve for the parameter $\bar{\phi}$:

$$\begin{aligned}
\bar{\phi}\frac{\sqrt{0.16}}{0.05} &= 0.40 \\
\bar{\phi} &= 0.05
\end{aligned}$$

The risk-neutral version of Choice D is:

$$\begin{aligned}
dr &= \left[a(r) + \bar{\phi}r\right]dt + \sigma(r)d\tilde{Z} \\
&= \left[0.3(0.125 - r) + 0.05r\right]dt + 0.05\sqrt{r}d\tilde{Z} \\
&= 0.25(0.15 - r)dt + 0.05\sqrt{r}d\tilde{Z}
\end{aligned}$$

Choice D has the same risk-neutral process as Ann's model, and therefore it will produce the same prices and yields as Ann's model. Therefore, Choice D is the correct answer.

For the sake of thoroughness, let's consider Choice E as well.

The Sharpe ratio for Choice E is the same as the Sharpe ratio we found for Choice C:

$$\bar{\phi}\frac{\sqrt{0.16}}{0.05} = 0.05$$
$$\bar{\phi} = 0.00625$$

The risk-neutral version of Choice E is:

$$\begin{aligned} dr &= \left[a(r) + \bar{\phi}r\right]dt + \sigma(r)d\tilde{Z} \\ &= \left[0.3(0.15 - r) + 0.00625r\right]dt + 0.05\sqrt{r}d\tilde{Z} \\ &= 0.29375(0.153191 - r)dt + 0.05\sqrt{r}d\tilde{Z} \end{aligned}$$

The risk-neutral version of Choice E is not the same as the risk-neutral version of Ann's model.

Solution 21

C Chapter 18, Estimating Parameters from Observed Data

The quickest way to do this problem is to input the data into a calculator.

Using the TI-30X IIS, the procedure is:

[2nd][STAT] (Select 1-VAR) [ENTER]

[DATA]

X1= $\ln(95/100)$ ↓↓ (Hit the down arrow twice)

X2= $\ln(105/95)$ ↓↓

X3= $\ln(111/105)$ ↓↓

X4= $\ln(103/111)$ [ENTER]

[STATVAR] → → (Arrow over to Sx)

$\times\sqrt{12}$ [ENTER] (The result is 0.290602821)

[STATVAR] → (Arrow over to $\bar{x}$)

$\times$ 12 [ENTER] (The result is 0.088676407)

To exit the statistics mode: [2nd] [EXITSTAT] [ENTER]

Alternatively, using the BA II Plus calculator, the procedure is:

[2nd][DATA] [2nd][CLR WORK]

X01= 95/100 = LN [ENTER] ↓↓ (Hit the down arrow twice)

X02= 105/95 = LN [ENTER] ↓↓

X03= 111/105 = LN [ENTER] ↓↓

X04= 103/111 = LN [ENTER]

[2nd][STAT] ↓↓↓ $\times \sqrt{12}$ = (The result is 0.29060282)

[2nd][STAT] ↓↓ $\times$ 12 = (The result is 0.08867641)

To exit the statistics mode: [2nd][QUIT]

Alternatively, using the TI-30XS MultiView, the procedure is:

[data] [data] 4 (to clear the data table)

(enter the data below)

L1	L2	L3
100	95	-----------
95	105	
105	111	
111	103	

(place cursor in the L3 column)

[data] → (to highlight FORMULA)

1 [ln] [data] 2 / [data] 1) [enter]

[2nd] [quit] [2nd] [stat] 1

DATA: (highlight L3) FRQ: (highlight one) (select CALC) [enter]

2 (to obtain $\bar{x}$)

$\bar{x} \times 12$ [enter] The result is 0.08867641

[2nd] [stat] 3

3 (to obtain Sx)

$Sx \times \sqrt{12}$ [enter] The result is 0.290602082

Therefore, the annualized estimates for the mean and standard deviation of the normal distribution are:

$$\hat{\alpha} - \delta - 0.5\hat{\sigma}^2 = 0.088676$$
$$\hat{\sigma} = 0.29060$$

The estimate for the annualized expected return is:

$$\hat{\alpha} = \frac{\bar{r}}{h} + \delta + 0.5\hat{\sigma}^2 = 0.088676 + 0 + 0.5 \times (0.29060)^2 = 0.13090$$

We know that the stock price at the end of month 4 is \$103. The expected value of the stock at the end of month 12 is:

$$E[S_T] = S_t e^{(\alpha-\delta)(T-t)}$$
$$E[S_1] = S_{\frac{4}{12}} e^{(0.13090-0)(1-4/12)} = 103e^{0.13090\times 2/3} = 112.3924$$

Solution 22

A Chapter 13, Market-Maker Profit

Question 47 of the Sample Exam uses a constant risk-free interest rate, but in the solution provided by the Society of Actuaries a "remark" describes how the question could still be answered if the risk-free interest rate is deterministic but not necessarily constant.

Let's assume that "several months ago" was time 0. Further, let's assume that the options expire at time T and that the current time is time t.

The interest rate is not necessarily constant, so it can vary over time. Therefore, we replace the usual discount factors as described below:

$$e^{-rT} \text{ is replaced by } e^{-\int_0^T r(s)ds}$$
$$e^{-r(T-t)} \text{ is replaced by } e^{-\int_t^T r(s)ds}$$

We can use put-call parity to obtain a system of 2 equations:

$$17.26 + K\ e^{-\int_0^T r(s)ds} = 100 + 6.84$$
$$8.41 + Ke^{-\int_t^T r(s)ds} = 90 + 10.49$$

This can be solved to find $e^{\int_t^T r(s)ds}$:

$$\left.\begin{aligned} Ke^{-\int_0^T r(s)ds} &= 89.58 \\ Ke^{-\int_t^T r(s)ds} &= 92.08 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} Ke^{-\int_0^t r(s)ds} e^{-\int_t^T r(s)ds} &= 89.58 \\ Ke^{-\int_t^T r(s)ds} &= 92.08 \end{aligned}\right\} \Rightarrow e^{\int_0^t r(s)ds} = \frac{92.08}{89.58}$$

The market-maker sold 100 of the put options. From the market-maker's perspective, the value of this position was equal to the quantity owned times the price:

$$-100 \times 6.84 = -684.00$$

The delta of the call option can be used to determine the delta of the put option:

$$\Delta_{Put} = \Delta_{Call} - e^{-\delta T} = 0.697 - e^{-0\times T} = 0.697 - 1 = -0.303$$

The delta of the position, from the perspective of the market-maker, was:

$$\text{Delta of a short position in 100 puts} = -100 \times (-0.303) = 30.3$$

To delta-hedge the position, the market-maker sold 30.3 shares of stock. The value of this position was the quantity owned times the price:

$$-30.3 \times 100 = -3,030.00$$

The proceeds from selling the put options and the stock were available to be lent at the risk-free rate. The value of this cash position at time 0 was:

$$684.00 + 3,030.00 = 3,714.00$$

The initial position, from the perspective of the market-maker, was:

Component	Value
Options	–684.00
Shares	–3,030.00
Risk-Free Asset	3,714.00
Net	0.00

After t years elapsed, the value of the options changed by:

$$-100 \times (10.49 - 6.84) = -365.00$$

After t years elapsed, the value of the shares of stock changed by:

$$-30.3 \times (90.00 - 100.00) = 303.00$$

After t years elapsed, the value of the funds that were lent at the risk-free rate changed by:

$$3,714.00\left(e^{\int_0^t r(s)ds} - 1\right) = 3,714.00\left(\frac{92.08}{89.58} - 1\right) = 103.65$$

The sum of these changes is the profit.

Component	Change
Gain on Options	–365.00
Gain on Stock	303.00
Interest	103.65
Overnight Profit	41.65

The change in the value of the position is \$41.65, and this is the profit.

Solution 23

E Chapter 19, Forward Price with Monte Carlo Valuation

To find the value of the option at time 0, we use the risk-neutral distribution. The Monte Carlo estimate for the current price of the contingent claim is:

$$\bar{V} = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} V_i(T)$$

$$= \frac{e^{-0.08\times 1}}{7}\left[\frac{1,000}{31.91} + \frac{1,000}{83.32} + \frac{1,000}{31.97} + \frac{1,000}{22.05} + \frac{1,000}{21.56} + \frac{1,000}{23.54} + \frac{1,000}{36.50}\right]$$

The forward price for an asset that does not pay dividends is its current price accumulated forward at the risk-free rate of return. Therefore, the forward price of the contingent claim is the expression above, accumulated at 8%:

$$\begin{aligned}
&\frac{e^{-0.08\times 1}}{7}\left[\frac{1{,}000}{31.91}+\frac{1{,}000}{83.32}+\frac{1{,}000}{31.97}+\frac{1{,}000}{22.05}+\frac{1{,}000}{21.56}+\frac{1{,}000}{23.54}+\frac{1{,}000}{36.50}\right]\times e^{0.08}\\
&=\frac{1}{7}\left[\frac{1{,}000}{31.91}+\frac{1{,}000}{83.32}+\frac{1{,}000}{31.97}+\frac{1{,}000}{22.05}+\frac{1{,}000}{21.56}+\frac{1{,}000}{23.54}+\frac{1{,}000}{36.50}\right]\\
&=33.75
\end{aligned}$$

The Monte Carlo estimate for the forward price of the contingent claim is 33.75.

The quickest way to answer this question is to use the TI-30XS MultiView Calculator:

[data] [data] 4 (to clear the data table)

(enter the data below)

L1	L2	L3
31.91	-----------	-----------
83.32		
31.97		
22.05		
21.56		
23.54		
36.50		

(place cursor in the L2 column)

[data] → (to highlight FORMULA) 1

1,000 / [data] 1) [enter]

[2nd] [quit] [2nd] [stat] 1

DATA: (highlight L2) FRQ: (highlight one) (select CALC) [enter]

$\bar{x}$ is 33.74731289.

Solution 24

D Chapter 12, Elasticity

The contingent claim can be replicated with a portfolio consisting of a long call with a strike price of \$40, a long put with a strike price of \$40, and borrowing the present value of \$10. The current value of the contingent claim is:

$$C(40)+P(40)-10e^{-0.09}$$

To find the value of this contingent claim, we begin by finding the value of the call option:

The first step is to calculate d_1 and d_2:

$$d_1 = \frac{\ln(S/K) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(43/40) + (0.09 - 0.04 + 0.5 \times 0.35^2) \times 1}{0.35\sqrt{1}}$$
$$= 0.52449$$
$$d_2 = d_1 - \sigma\sqrt{T} = 0.52449 - 0.35\sqrt{1} = 0.17449$$

We have:

$$N(d_1) = N(0.52449) = 0.70003$$

$$N(d_2) = N(0.17449) = 0.56926$$

The value of the European call option is:

$$C(40) = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$$
$$= 43e^{-0.04\times1} \times 0.70003 - 40e^{-0.09\times1} \times 0.56926 = 8.1104$$

The value of the European put option is:

$$P(40) = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$
$$= 40e^{-0.09\times1} \times (1 - 0.56926) - 43e^{-0.04\times1} \times (1 - 0.70003) = 3.3537$$

The current value of the contingent claim is:

$$V = C(40) + P(40) - 10e^{-0.09} = 8.1104 + 3.3537 - 10e^{-0.09} = 2.3248$$

The delta of the call option is:

$$\Delta_{Call} = e^{-\delta T}N(d_1) = e^{-0.04\times1} \times 0.70003 = 0.67258$$

The delta of the put option is:

$$\Delta_{Put} = \Delta_{Call} - e^{-\delta T} = 0.67258 - e^{-0.04\times1} = -0.28821$$

The delta of the contingent claim is the delta of the call plus the delta of the put minus the delta of the loan (i.e., zero):

$$\Delta = 0.67258 + (-0.28821) + 0.0000 = 0.38437$$

The elasticity of the contingent claim is:

$$\Omega = \frac{S\Delta}{V} = \frac{43 \times (0.38437)}{2.3248} = 7.1093$$

Solution 25

E Chapter 9, Strike Price Grows Over Time

The prices of the options decrease as time to maturity increases. Therefore, if the strike price increases at a rate that is greater than the risk-free rate, then arbitrage is available.

Option C expires 0.5 years after Option B, so let's accumulate Option B's strike price for 0.5 years at the risk-free rate:

$$53e^{0.13\times0.5} = 56.5594$$

Since the strike price of Option C is \$57, the strike price grows from time 1.5 to time 2.0 at a rate that is greater than the risk-free rate of return. Consequently, arbitrage can be earned by purchasing Option C and selling Option B.

The arbitrageur buys the 2-year option for \$3.00 and sells the 1.5-year option for \$3.50 The difference of \$0.50 is lent at the risk-free rate of return.

<u>The 1.5-year option</u>

After 1.5 years, the stock price is \$52.00. Therefore, the 1.5-year option is exercised against the arbitrageur. The arbitrageur borrows \$53 and uses it to buy a share of stock. As a result, at the end of 2 years the arbitrageur owns the share of stock and owes the accumulated value of the \$53. This position results in the following cash flow at the end of 2 years:

$$58.00 - 53e^{0.13\times0.5} = 1.4406$$

<u>The 2-year option</u>

The stock price of \$58.00 at the end of 2 years is greater than the strike price of the 2-year option, which is \$57. Therefore, the 2-year put option expires worthless, and the resulting cash flow is zero.

<u>The net cash flow</u>

The net cash flow at the end of 2 years is the sum of the accumulated value of the \$0.50 that was obtained by establishing the position, the \$1.4406 resulting from the 1.5-year option, and the \$0.00 resulting from the 2-year option:

$$0.50e^{0.13\times2} + 1.4406 + 0.00 = 2.0890$$

Solution 26

B Chapter 11, Expected Return

The up and down factors are constant throughout the tree, and therefore the risk-neutral probability of an upward movement is also constant and can be calculated using any node. Below we use the first node:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.10-0.00)(1)} - \frac{56}{80}}{\frac{104}{80} - \frac{56}{80}} = 0.67528$$

Working from right to left, we create the tree of prices for the European put option:

Stock			European Put		
		135.20			0.0000
	104.00			7.9918	
80.00		72.80	15.0150		27.2000
	56.00			34.4837	
		39.20			60.8000

The realistic probability is also constant throughout the tree, so as with the risk-neutral probability, we can use the values at the first node to calculate it:

$$p = \frac{e^{(\alpha-\delta)h} - d}{u-d} = \frac{e^{(0.15-0.00)(1)} - \frac{56}{80}}{\frac{104}{80} - \frac{56}{80}} = 0.76972$$

When the stock price is \$80.00, we have:

$$e^{\gamma h} = \frac{\left[(p)V_u + (1-p)V_d\right]}{V}$$

$$e^{\gamma} = \frac{(0.76972)(7.9918) + (1-0.76972)(34.4837)}{15.015}$$

$$\gamma = \ln\left(\frac{(0.76972)(7.9918) + (1-0.76972)(34.4837)}{15.015}\right) = -0.0634$$

When the stock price is \$56.00, we have:

$$e^{\gamma h} = \frac{\left[(p)V_u + (1-p)V_d\right]}{V}$$

$$e^{\gamma} = \frac{(0.76972)(0.00) + (1-0.76972)(27.20)}{7.9918}$$

$$\gamma = \ln\left(\frac{(0.76972)(0.00) + (1-0.76972)(27.20)}{7.9918}\right) = -0.2437$$

When the stock price is \$39.20, we have:

$$e^{\gamma h} = \frac{\left[(p)V_u + (1-p)V_d\right]}{V}$$

$$e^{\gamma} = \frac{(0.76972)(27.20) + (1-0.76972)(60.80)}{34.4837}$$

$$\gamma = \ln\left(\frac{(0.76972)(27.20) + (1-0.76972)(60.80)}{34.4837}\right) = 0.0131$$

The tree of values for γ is filled in below:

		N/A
	–0.2437	
–0.0634		N/A
	0.0131	
		N/A

The highest of the three values above is 1.31%.

Solution 27

A Chapter 24, Rendleman-Bartter Model

In the Rendleman-Bartter Model, the short-term rate follows geometric Brownian motion. From Chapter 18 of the Derivatives Markets textbook, the payoff of a call option can be found as:

$$E\left[\text{Call Payoff}\right] = S_t e^{(\alpha-\delta)(T-t)} N(\hat{d}_1) - K \times N(\hat{d}_2)$$

We can consider the derivative to be 1,000 call options with the short rate replacing the stock price and 0.11 replacing the strike price:

$$\hat{d}_1 = \frac{\ln\left(\frac{r(t)}{K}\right) + (\alpha - \delta + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln\left(\frac{0.08}{0.11}\right) + (0.12 + 0.5 \times 0.20^2) \times 1}{0.20\sqrt{1}} = -0.89227$$

$$\hat{d}_2 = \frac{\ln\left(\frac{r(t)}{K}\right) + (\alpha - \delta - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln\left(\frac{0.08}{0.11}\right) + (0.12 - 0.5 \times 0.20^2) \times 1}{0.20\sqrt{1}} = -1.09227$$

The values of $N(\hat{d}_1)$ and $N(\hat{d}_2)$ are:

$$N(\hat{d}_1) = N(-0.89227) = 0.18612$$

$$N(\hat{d}_2) = N(-1.09227) = 0.13736$$

The expected value of the payoff is:

$$E\left[\text{Payoff}\right] = 1{,}000 \times \left[S_t e^{(\alpha-\delta)(T-t)} N(\hat{d}_1) - K \times N(\hat{d}_2)\right]$$
$$= 1{,}000 \times \left[0.08e^{0.12(1)} \times 0.18612 - 0.11 \times 0.13736\right] = 1.6784$$

Solution 28

C Chapter 12, Black-Scholes Formula Using Prepaid Forward Prices

The Black-Scholes formula for the call option on Stock Y is:

$$C_{Eur}(S,K,\sigma,r,T,\delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$
$$16.30 = 80e^{-4\delta} N(d_1) - Ke^{-0.07\times 4} N(d_2) = 80e^{-4\delta} N(d_1) - Ke^{-0.28} N(d_2)$$

Consider the price of the call option on Stock Z. To avoid confusion, we use $\hat{d}_1$ and $\hat{d}_2$ for the call option on Stock Z to distinguish from d_1 and d_2 above:

$$C_{Eur}^{Z} = 40e^{-(0.25\delta)\times 16} N(\hat{d}_1) - \left(0.5Ke^{0.84}\right) e^{-0.07\times 16} N(\hat{d}_2)$$
$$= 40e^{-4\delta} N(\hat{d}_1) - 0.5Ke^{-0.28} N(\hat{d}_2)$$
$$= 0.5\left[80e^{-4\delta} N(\hat{d}_1) - Ke^{-0.28} N(\hat{d}_2)\right]$$

Let's calculate d_1 and d_2 for the call on Stock Y:

$$d_1 = \frac{\ln\left(\dfrac{F_{0,T}^P(S)}{F_{0,T}^P(K)}\right) + 0.5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln\left(\dfrac{80e^{-4\delta}}{Ke^{-0.07\times 4}}\right) + 0.5\sigma^2 \times 4}{\sigma\sqrt{4}} = \frac{\ln\left(\dfrac{80e^{-4\delta}}{Ke^{-0.28}}\right) + 2\sigma^2}{2\sigma}$$
$$d_2 = d_1 - \sigma\sqrt{T} = d_1 - 2\sigma$$

Let's calculate $\hat{d}_1$ and $\hat{d}_2$ for the call on Stock Z:

$$\hat{d}_1 = \frac{\ln\left(\dfrac{F_{0,T}^P(S)}{F_{0,T}^P(K)}\right) + 0.5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln\left(\dfrac{40e^{-0.25\delta\times 16}}{0.5Ke^{0.84}e^{-0.07\times 16}}\right) + 0.5(0.5\sigma)^2(16)}{(0.5\sigma)\sqrt{16}}$$
$$= \frac{\ln\left(\dfrac{40e^{-4\delta}}{0.5Ke^{-0.28}}\right) + 2\sigma^2}{2\sigma} = \frac{\ln\left(\dfrac{80e^{-4\delta}}{Ke^{-0.28}}\right) + 2\sigma^2}{2\sigma} = d_1$$
$$\hat{d}_2 = \hat{d}_1 - 0.5\sigma\sqrt{16} = d_1 - 2\sigma = d_2$$

From the above, we see that:

$$\hat{d}_1 = d_1 \quad \text{and} \quad \hat{d}_2 = d_2$$

This implies that the price of the call on Stock Z is half the price of the call on Stock Y:

$$C_{Eur}^{Z} = 0.5\left[80e^{-4\delta}N(\hat{d}_1) - Ke^{-0.28}N(\hat{d}_2)\right] = 0.5[16.30] = 8.15$$

Solution 29

B Chapter 14, Compound Options

The annual effective risk-free rate of return is 25%:

$$e^{r} = 1.25$$

If the strike prices of the compound options are based on the following formula, then we will be able to draw some conclusions about whether the compound options expire in the money:

$$x = D - K\left(1 - e^{-r(T-t_1)}\right) = 20 - K\left(1 - 1.25^{-(T-0)}\right)$$

These values of x are calculated for the four American call options below:

Strike Price	Remaining Years Until Maturity	x	Exercise Now?
62	1	7.60	NO
44	2	4.16	YES
56	1	8.80	YES
50	2	2.00	NO

Observe that these values of x correspond to the strike prices of Ethel's compound options. This means that we can use the following rules for the compound options:

Exercise American call option at time t_1 $\Leftrightarrow$ Don't exercise call on put at time t_1

Don't exercise American call option at time t_1 $\Leftrightarrow$ Exercise call on put at time t_1

Since exercising a call on a put implies not exercising the corresponding put on a put, and vice versa, we can add some implications to the right of the rules above:

Exercise American call option at time t_1 $\Leftrightarrow$ Don't exercise call on put at time t_1 $\Leftrightarrow$ Exercise put on put at time t_1

Don't exercise American call option at time t_1 $\Leftrightarrow$ Exercise call on put at time t_1 $\Leftrightarrow$ Don't exercise put on put at time t_1

The first American call is not exercised, so Option A, which is a CallOnPut, is exercised.

The second American call is exercised, so Option B, which is a PutOnPut, is exercised.

The third American call is exercised, so Option C, which is a CallOnPut is not exercised.

The fourth American call is not exercised, so Option D, which is a PutOnPut is not exercised.

The options that are exercised upon maturity are the only ones that are worth more than zero. Therefore Options A and B are the only options worth more than zero.

Solution 30

B Chapter 20, Drift

We can use the theta of the claim to determine β:

$$\frac{\partial V(t)}{\partial t} = 0 \qquad t \geq 0$$

$$\frac{\partial \left\{ \left[S(t) \right]^3 + \beta t \right\}}{\partial t} = 0 \qquad t \geq 0$$

$$\beta = 0 \qquad t \geq 0$$

The value of the claim is therefore:

$$V(t) = \left[S(t) \right]^3$$

We have:

$$V_S = 3S^2 \qquad V_{SS} = 6S \qquad V_t = 0$$

From Itô's Lemma:

$$\begin{aligned} dV &= V_S dS + 0.5 V_{SS} (dS)^2 + V_t dt \\ &= 3S^2 \left(0.08Sdt + 0.20SdZ\right) + 0.5\left(6S\right)\left(0.08Sdt + 0.20SdZ\right)^2 + 0 \\ &= S^3 \left(0.24dt + 0.60dZ\right) + 3S\left(0.04S^2 dt\right) \\ &= 0.36S^3 dt + 0.60S^3 dZ \\ &= 0.36Vdt + 0.60VdZ \end{aligned}$$

The drift is the expected change per unit of time in the price of the claim. From the stochastic differential equation above, we observe that the drift is $0.36V(t)$.