Chapter 11: Loans

Solution 11.01

A Section 11.02, Level Payment Amortized Loans

After the 15th payment, the remaining balance is equal to the present value of the remaining 30 payments:

$$400a_{\overline{30}|}$$

After subtracting out the extra 2,000, the remaining balance is:

$$400a_{\overline{30}|} - 2,000$$

This balance is then paid off with 25 payments, so the time-15 equation of value is:

$$400a_{\overline{30}|} - 2,000 = X \times a_{\overline{25}|}$$

We can now solve for the revised annual payment:

$$400 \times \frac{1-1.07^{-30}}{0.07} - 2,000 = Pmt \times \frac{1-1.07^{-25}}{0.07}$$

$$400 \times 12.4090 - 2,000 = Pmt \times 11.6536$$

$$Pmt = \mathbf{254.31}$$

The BA-II Plus calculator can be used to solve this problem as follows:

45 [–] 15 [=] [*N*] 7 [*I*/*Y*] 400 [*PMT*] [CPT] [*PV*]

[+] 2,000 [=] [*PV*] 25 [*N*]

[CPT] [*PMT*]

Result is **254.31**.

Solution 11.02

A Section 11.02, Level Payment Amortized Loans

The payment of 14 is exactly equal to the interest on the loan each quarter:

$$700 \times \frac{0.08}{4} = 14$$

Since the payment does not exceed the interest, the amount of principal paid down is zero for each payment:

$$Prn_t = Pmt_t - Int_t = 14 - 14 = \mathbf{0}$$

Solution 11.03

E Section 11.02, Level Payment Amortized Loans

The amount of each level payment is:

$$Pmt = 73.09 + 426.91 = 500$$

The principal repaid in the 17th payment is found below:

$$\frac{Prn_{t+k}}{Prn_t} = (1+i)^k$$

$$\frac{Prn_{17}}{73.09} = (1.06)^{12}$$

$$Prn_{17} = 147.0714$$

The interest paid in the 17th payment is the payment minus the principal paid:

$$500 - 147.0714 = \mathbf{352.93}$$

Solution 11.04

E Section 11.02, Level Payment Amortized Loans

The interest paid on the first loan is:

$$4,000 \times \left(1.04^{10} - 1\right) = 1,920.9771$$

The interest paid on the second loan is:

$$4,000 \times 10 \times 0.04 = 1,600$$

The interest paid on the third loan is the sum of the 10 payments minus the total principal paid. The total principal paid is equal to the initial principal of 4,000:

$$\frac{4,000}{a_{\overline{10}|0.04}} \times 10 - 4,000 = \frac{4,000}{8.1109} \times 10 - 4,000 = 493.1638 \times 10 - 4,000$$

$$= 931.6378$$

The total amount of interest paid on all 3 loans is:

$$1,920.9771 + 1,600 + 931.6378 = \mathbf{4,452.6149}$$

Solution 11.05

B Section 11.02, Level Payment Amortized Loans

The first 5 payments pay the principal down at a rate that is equal to 100% of the interest rate. Since the interest rate is 10%, the portion of the principal that is paid down by each of the first 10 payments is:

$$(200\% - 100\%) \times 0.10 = 0.10$$

After 5 years, the original principal has been reduced by 10% per year for 5 years. The equation of value at the end of 5 years is:

$$10,000 \times 0.90^5 = Xa_{\overline{5}|0.10}$$

$$5,904.90 = X \times \frac{1 - 1.10^{-5}}{0.10}$$

$$X = \mathbf{1,557.70}$$

The BA-II Plus can be used to answer this question:

10,000 [×] 0.9 [y^x] 5 [=] [*PV*]

5 [*N*] 10 [*I/Y*] [CPT] [*PMT*]

Result is –1,557.70. Solution is **1,557.70**.

Solution 11.06

E Section 11.02, Level Payment Amortized Loans

The following formulas are useful for answering this question:

$$Int_t = (1 - v^{n-t+1})Pmt$$

$$Prn_t = v^{n-t+1} \times Pmt$$

Substituting 1 for the level payment amount, the sum of the principal paid in year t and the interest paid in year $(t+1)$ is:

$$X = Prn_t + Int_{t+1} = v^{n-t+1} + \left(1 - v^{n-(t+1)+1}\right) = 1 + v^{n-t+1} - v^{n-t}$$
$$= 1 + v^{n-t}(v-1) = 1 - v^{n-t}(1-v) = 1 - v^{n-t}d$$

The answer is Choice **E**.

Solution 11.07

C Section 11.02, Level Payment Amortized Loans

The final payment is the loan balance at the end of year 4 accrued with interest:

$$Pmt = 911.74 \times 1.07 = 975.5618$$

The initial loan balance is:

$$Pmt \times a_{\overline{5}|} = 975.5618 \times \frac{1 - 1.07^{-5}}{0.07} = 3{,}999.9960$$

The first principal payment is equal to the level payment minus the interest on the initial loan balance:

$$975.5618 - 3{,}999.9960 \times 0.07 = \mathbf{695.56}$$

Solution 11.08

D Section 11.02, Level Payment Amortized Loans

The principal payment increases by (1 + i) each year:

$$\frac{Prn_{t+k}}{Prn_t} = (1+i)^k$$

$$\frac{Prn_6}{Prn_1} = (1.07)^5$$

$$\frac{1{,}015.13}{Prn_1} = (1.07)^5$$

$$Prn_1 = \mathbf{723.77}$$

Solution 11.09

C Section 11.02, Level Payment Amortized Loans

The monthly effective interest rate is:

$$\frac{0.10}{12} = 0.008333$$

The initial loan payment is:

$$Pmt = \frac{500{,}000}{a_{\overline{15\times 12}|0.008333}} = \frac{500{,}000}{\frac{1 - 1.008333^{-180}}{0.008333}} = \frac{500{,}000}{93.0574} = 5{,}373.0256$$

The balance after the 48th payment can be found using the prospective method. At the new interest rate, the smaller payments pay off the balance in 11 years:

$$5{,}373.0256 \times a_{\overline{132}|0.008333} = (5{,}373.0256 - 473.98)a_{\overline{132}|i/12}$$

$$5{,}373.0256 \times 79.8730 = 4{,}899.0456 \times a_{\overline{132}|i/12}$$

$$429{,}159.5981 = 4{,}899.0456 \times a_{\overline{132}|i/12}$$

The easiest way to find i is to use the BA II Plus calculator. Let's use the calculator from the beginning of this question:

180 [N] 10 [$\div$] 12 [=] [I/Y] 500,000 [+/-] [PV] [CPT] [PMT]

Result is 5,373.0256.

11 [$\times$] 12 [=] [N] [CPT] [PV]

Result is –429,159.5981.

[RCL] [PMT] [–] 473.98 [=] [PMT] [CPT] [I/Y] [$\times$] 12 [=]

Result is 8.00. Answer is **8.00%.**

Solution 11.10

B Section 11.02, Level Payment Amortized Loans

The monthly effective rate at which the loan is originally made is:

$$\frac{0.09}{12} = 0.0075$$

The original payment amount is:

$$\frac{75{,}000}{a_{\overline{240}|0.0075}} = 674.7945$$

After the 24th payment, the remaining balance can be found using the prospective method:

$$L_{24} = 674.7945 \times a_{\overline{240-24}|0.0075} = 72{,}059.1797$$

The new payment amount is the amount needed to pay off the remaining balance at the new interest rate of 7% compounded monthly:

$$\frac{72{,}059.1797}{a_{\overline{240-24}|\frac{0.07}{12}}} = \frac{72{,}059.1797}{a_{\overline{216}|0.005833}} = \mathbf{587.64}$$

Alternatively, we can use the BA II Plus to answer this question:

240 [N] 9 [$\div$] 12 [=] [I/Y] 75,000 [+/-] [PV] [CPT] [PMT]

Result is 674.7945.

240 [–] 24 [=] [N] [CPT] [PV]

Result is –72,059.1797.

7 [$\div$] 12 [=] [I/Y] [CPT] [PMT]

Answer is **587.64**.

Solution 11.11

D Section 11.02, Level Payment Amortized Loans

The annual effective interest rate is:

$$i = \frac{0.06}{1-0.06} = 0.06383$$

We can solve for the amount of the 5 level payments:

$$24{,}000 = Pmt \times a_{\overline{5}|0.06383}$$

$$24{,}000 = Pmt \times 4.1688$$

$$Pmt = 5{,}757.0013$$

If the first 4 payments were instead 5,800, then the balance at the end of 4 years would be:

$$\frac{24,000}{0.94^4} - 5,800 \times s_{\overline{4}|0.06383} = \frac{24,000}{0.94^4} - 5,800 \times 4.3995$$
$$= 5,222.4070$$

The final payment is the balance at the end of 4 years, accumulated for one additional year of interest:

$$\frac{5,222.4070}{0.94} = \mathbf{5,555.75}$$

We can use the BA II Plus to answer this question:

0.06 [÷] 0.94 [×] 100 [=] [*I/Y*]

5 [*N*] 24,000 [+/-] [*PV*] [CPT] [*PMT*]

Result is 5,757.0013.

5,800 [*PMT*] 4 [*N*] [CPT] [*FV*]

[÷] 0.94 [=]

Answer is **5,555.75**.

Solution 11.12

D Section 11.02, Level Payment Amortized Loans

The amount of the equal annual payments under option (i) is:

$$\frac{10,000}{a_{\overline{20}|0.05}} = \frac{10,000}{12.4622} = 802.4259$$

Alternatively, the amount of the equal annual payments under option (i) can be found using the BA-II Plus calculator:

20 [*N*] 5 [*I/Y*] 10,000 [+/-] [*PV*] [CPT] [*PMT*]

Result is 802.4259.

The sum of the payments under option (i) is:

$$802.4259 \times 20 = 16,048.5174$$

Since the payments of 500 under option (ii) are over and above the payment of the interest, the balance of the loan decreases by 500 per year. Therefore, the interest payments decline each year. The sum of the payments under option (ii) is:

$$500 \times 20 + i(10,000 + 9,500 + \cdots + 500) = 10,000 + 500i(20 + 19 + \cdots + 1)$$
$$= 10,000 + 500i \times \frac{20 \times 21}{2} = 10,000 + 105,000i$$

Setting the sum of the payments under option (i) equal to the sum of the payments under option (ii) allows us to solve for i:

$$16,048.5174 = 10,000 + 105,000i$$
$$i = \frac{6,048.5174}{105,000}$$
$$i = \mathbf{0.0576}$$

Solution 11.13

C Section 11.02, Level Payment Amortized Loan

We begin by finding the level payment amount:

$$Prn_t = v^{n-t+1} \times Pmt$$

$$1{,}370.65 = \frac{Pmt}{1.065^{10-6+1}}$$

$$Pmt = 1{,}877.9093$$

The initial value of the loan can be found using the prospective method:

$$1{,}877.9093 \times a_{\overline{10|}} = 1{,}877.9093 \times \frac{1-1.065^{-10}}{0.065} = 1{,}877.9093 \times 7.1888$$
$$= 13{,}499.9710$$

The total amount of interest paid on the loan is equal to the total amount of the payments minus the initial loan balance:

$10 \times 1{,}877.9093 - 13{,}499.9710 = 18{,}779.0929 - 13{,}499.9710 =$ **5,279.12**

The BA II Plus can be used to answer this question:

1,370.65 [×] 1.065 [y^x] 5 [=]

Result is 1,877.9093.

[*PMT*] 10 [*N*] 6.5 [*I/Y*] [CPT] [*PV*]

Result is –13,499.9710.

[+] 10 [×] [RCL] [*PMT*] [=]

Answer is **5,279.12**.

Solution 11.14

B Section 11.02, Level Payment Amortized Loan

We make use of the following formula for the interest portion of a payment:

$$Int_t = (1 - v^{n-t+1})Pmt$$

The formula can be used to find the following expressions:

$$Int_{n-2} = (1 - v^3)Pmt$$

$$Int_{n-5} = (1 - v^6)Pmt$$

$$Int_1 = (1 - v^n)Pmt$$

The interest portion of the payment at time (n – 2) is equal to 0.5471 of the interest portion of the payment at time (n – 5), which allows us to solve for v:

$$(1 - v^3)Pmt = 0.5471(1 - v^6)Pmt$$

$$1 = 0.5471(1 + v^3)$$

$$v = 0.9390$$

The interest portion of the payment at time (n – 2) is equal to 0.2209 of the interest portion of the first payment, which allows us to solve for n:

$$(1 - v^3)Pmt = 0.2209(1 - v^n)Pmt$$

$$0.1722 = 0.2209(1 - v^n)$$

$$0.2205 = v^n$$

$$\ln(0.2205) = n \times \ln(v)$$

$n =$ **24.00**

Solution 11.15

E Section 11.02, Level Payment Amortized Loan

We do not need to know the details of the payments occurring after the 16th year to answer this question.

The principal repaid in year 16 is the payment of 1,500 minus the interest on the outstanding balance at the end of 15 years:

$$X = 1,500 - iL_{15}$$

The interest paid in the first year is the interest rate times the initial loan balance:

$$\begin{aligned} i \times L_0 &= i\left(1,500 \times a_{\overline{15|}} + v^{15}L_{15}\right) = 1,500(1 - v^{15}) + iv^{15}L_{15} \\ &= 1,500 - 1,500v^{15} + iv^{15}L_{15} = 1,500 - v^{15}\left(1,500 - iL_{15}\right) \\ &= 1,500 - Xv^{15} \end{aligned}$$

The final expression above matches Choice **E**.

Solution 11.16

E Section 11.02, Level Payment Amortized Loan

We do not need to know the details of the payments occurring after the 11th year to answer this question.

The principal repaid in year 11 is the payment of 1,000 minus the interest on the outstanding balance at the end of 10 years:

$$X = 1,000 - iL_{10}$$

The interest paid in the first year is the interest rate times the initial loan balance:

$$\begin{aligned} i \times L_0 &= i\left(1,000 \times a_{\overline{10|}} + v^{10}L_{10}\right) = 1,000(1 - v^{10}) + iv^{10}L_{10} \\ &= 1,000 - 1,000v^{10} + iv^{10}L_{10} = 1,000 - v^{10}\left(1,000 - iL_{10}\right) \\ &= 1,000 - Xv^{10} \end{aligned}$$

The final expression above matches Choice **E**.

Solution 11.17

C Section 11.02, Level Payment Amortized Loan

We use the following formulas:

$$Int_t = (1 - v^{n-t+1})Pmt$$

$$Prn_t = v^{n-t+1} \times Pmt$$

Using the information provided in the question, we have:

$$Int_1 = (1 - v^{20})Pmt = 4,316$$

$$Prn_{11} = v^{10} \times Pmt = 4,080$$

Dividing the first equation by the second equation allows us to solve for v:

$$\frac{(1-v^{20})Pmt}{v^{10}\times Pmt}=\frac{4,316}{4,080}$$

$$1-v^{20}=\frac{4,316}{4,080}v^{10}$$

$$v^{20}+\frac{4,316}{4,080}v^{10}-1=0$$

We can use the quadratic formula to solve for v^{10}:

$$v^{10}=\frac{-\frac{4,316}{4,080}\pm\sqrt{\left(\frac{4,316}{4,080}\right)^2-4\times 1\times(-1)}}{2}$$

$$v^{10}=0.6023 \quad \text{or} \quad v^{10}=-1.1660$$

We use the positive value of v^{10} to solve for i:

$$v^{10}=0.6023$$

$$i=0.05200$$

Since the interest paid in the first year is 4,316, we have:

$$Xi=4,316$$

$$X\times 0.05200=4,316$$

$$X=\mathbf{83,000.30}$$

The answer is Choice **C**.

Alternatively, we can use the answer choices provided to quickly consider each possibility. We use the calculator to divide the 1st year's interest by the possible value of L and then see if it produces the correct principal payment in the 11th year. We use the BA-II Plus:

20 [*N*]

4,316 [÷] 81,000 [×] 100 [=] [*I*/*Y*] 81,000 [*PV*] [CPT] [*PMT*]

[2nd] [AMORT] 11 [ENTER] ↓ [ENTER] 11 ↓↓

(Result is –3,975.97, so Choice A is not correct.)

[2nd] [QUIT]

4,316 [÷] 81,500 [×] 100 [=] [*I*/*Y*] 81,500 [*PV*] [CPT] [*PMT*]

[2nd] [AMORT] ↓↓↓

(Result is –4,001.98, so Choice B is not correct.)

[2nd] [QUIT]

4,316 [÷] 83,000 [×] 100 [=] [*I*/*Y*] 83,000 [*PV*] [CPT] [*PMT*]

[2nd] [AMORT] ↓↓↓

(Result is –4,079.98. This rounds to –4,080 so **Choice C is correct**.)

[2nd] [QUIT]

Solution 11.18

A Section 11.03, Drop Payments

If we accumulate the initial loan balance to time 3, then we can treat the loan as a loan with the first payment occurring one year later:

$$\frac{84,000}{(1-0.06)^3}=101,133.6602$$

The annual effective interest rate is:

$$\frac{0.06}{1-0.06} = 0.06383$$

We can solve the time-3 equation of value for n:

$$101{,}133.6602 = 10{,}000 \times a_{\overline{n|}}$$

$$101{,}133.6602 = 10{,}000 \times \frac{1-1.06383^{-n}}{0.06383}$$

$$0.3545 = 1.06383^{-n}$$

$$n = 16.7618$$

Therefore, there are 16 payments of 10,000 and a final drop payment at time 17:

$$101{,}133.6602 = 10{,}000 a_{\overline{16|}} + DropPmt \times 0.94^{17}$$

$$101{,}133.6602 = 10{,}000 \times \frac{1-1.06383^{-16}}{0.06383} + DropPmt \times 0.94^{17}$$

$$DropPmt = \mathbf{7{,}673.79}$$

We can use the BA II Plus calculator to answer this question:

84,000 [÷] 0.94 [y^x] 3 [=] [+/-] [*PV*]
0.06 [÷] 0.94 [×] 100 [=] [*I/Y*] 10,000 [*PMT*]
[CPT] [*N*]
Result is 16.7618.
16 [*N*] [CPT] [*FV*] [÷] 0.94 [=]
Answer is **7,673.79**.

Solution 11.19

D Section 11.03, Drop Payments

The monthly effective interest rate is:

$$\frac{0.09}{12} = 0.0075$$

The level payments satisfy the following time-0 equation of value:

$$400{,}000 = Pmt \times a_{\overline{240|}0.0075}$$

$$400{,}000 = Pmt \times 111.1450$$

$$Pmt = 3{,}598.9038$$

When we subtract the present value of the extra payments, the new equation of value is:

$$400{,}000 - 15{,}000 \times a_{\overline{4|}1.0075^{12}-1} = 3{,}598.9038 \times a_{\overline{n|}0.0075}$$

$$400{,}000 - 15{,}000 \times a_{\overline{4|}0.09381} = 3{,}598.9038 \times a_{\overline{n|}0.0075}$$

$$400{,}000 - 15{,}000 \times 3.2128 = 3{,}598.9038 \times \frac{1-1.0075^{-n}}{0.0075}$$

$$97.7541 = \frac{1-1.0075^{-n}}{0.0075}$$

$$0.2668 = 1.0075^{-n}$$

$$n = 176.8050$$

The final payment therefore occurs after 177 months, which is 14.75 years:

$$\frac{177}{12} = 14.75$$

The loan originated on January 1, 2011, and the final payment is made 14 years and 9 months later. Adding 14 years to January 1, 2011 brings us to January 1, 2025. Adding 9 more months brings us to October 1, 2025. The payments are made at the end of each month though, so the drop payment is made on **September 30, 2025**.

We can use the BA II Plus to answer this question:

400,000 [*PV*] 240 [*N*] 0.75 [*I*/*Y*] [CPT] [*PMT*]

Result is –3,598.9038.

[STO] 1

4 [*N*] 1.0075 [y^x] 12 [–] 1 [=] [×] 100 [=] [*I*/*Y*] 15,000 [*PMT*] [CPT] [*PV*]

[+] 400,000 [=]

Result is 351,807.5102.

[*PV*] 0.75 [*I*/*Y*] [RCL] 1 [*PMT*] [CPT] [*N*]

Result is 176.8050, so the final payment occurs after 177 months.

177 [÷] 12 [=]

Result is 14.75.

[+] 2011 [=]

Result is 2025.75.

The date of 2025.75 is 9 months after the beginning of 2025, so the drop payment is made on **September 30, 2025**.

Solution 11.20

B Section 11.04, Sinking Funds

The interest payment made at the end of each year is the interest rate of 11% times the loan amount of 100,000:

$$100,000 \times 0.11 = 11,000$$

The sinking fund payments accumulate to 100,000 at the end of 10 years. The sinking fund payment is found below:

$$SFP \times s_{\overline{10}|0.07} = 100,000$$

$$SFP \times \frac{1.07^{10} - 1}{0.07} = 100,000$$

$$SFP = \frac{100,000}{13.8164}$$

$$SFP = 7,237.7503$$

The total of the payments made over the 10-year period is equal to 10 times the sum of the interest payment and the sinking fund payment:

$$10 \times (7,237.7503 + 11,000) = \mathbf{182,377.50}$$

The BA-II Plus calculator can be used to solve this question as follows:

10 [*N*] 7 [*I*/*Y*] 100,000 [*FV*]

[CPT] [*PMT*] [+/-]

[+] 100,000 [×] 0.11 [=]

[×] 10 [=]

Result is **182,377.50**.

Solution 11.21

A Section 11.04, Sinking Funds

The deposits to the sinking fund are equal to 1,604.85 minus the interest on the loan:

$$1,604.85 - 20,000 \times 0.05 = 604.85$$

The accumulated value of the deposits is:

$$604.85 \times s_{\overline{20}|0.06} = 604.85 \times \frac{1.06^{20} - 1}{0.06} = 604.85 \times 36.7856 = 22,249.7648$$

After repaying the loan, the balance is:

$$22,249.7648 - 20,000 = \mathbf{2,249.76}$$


The BA-II Plus can be used to answer this question:

20 [*N*] 6 [*I/Y*] 1,604.85 [–] 20,000 [×] 0.05 [=] [*PMT*] [CPT] [*FV*]

[+/-] [–] 20,000 [=]

Solution is **2,249.76**.

Solution 11.22

A Section 11.04, Sinking Funds


We can use the BA-II Plus calculator to find the amount of the annual payment:

10 [*N*] 7 [*I/Y*] 10,000 [*PV*] [CPT] [*PMT*]

Result is –1,423.7750.

We reduce the payment amount by the interest that is paid to the lender in order to obtain the sinking fund payment that must accumulate to 10,000 at the end of 10 years. We can use the calculator to find the interest rate that the sinking fund must earn. Continuing from the calculation above, the keystrokes are:


[+] 0.055 [×] 10,000 [=]

Result is –873.7750, indicating that the sinking fund payment is 873.7750.

[*PMT*] 0 [*PV*] 10,000 [*FV*] [CPT] [*I/Y*]

Result is 2.9634.

The solution is **2.96%.**

Solution 11.23

B Section 11.04, Sinking Funds

Since the borrower pays the interest on the loan each year, the amount needed to pay off the loan at the end of 10 years is the original amount of 10,000. Therefore, the sinking fund payments must accumulate to 10,000.

$$10,000 \times 2i \times s_{\overline{10}|0.6i} = 10,000$$

$$20,000i \times \frac{(1 + 0.6i)^{10} - 1}{0.6i} = 10,000$$

$$(1 + 0.6i)^{10} - 1 = 0.3$$

$$0.6i = 0.2658$$

$$i = \mathbf{0.0443}$$

Solution 11.24

C Section 11.04, Sinking Funds

The sinking fund payment is:

$$SFP = \frac{L_0}{s_{\overline{14}|0.062}}$$

The amount owed to the lender is remains constant at L_0 until the loan is paid off, so the equation of value at the end of 6 years is:

$$L_0 - \frac{L_0}{s_{\overline{14}|0.062}} \times s_{\overline{6}|} = 59,053$$

$$L_0 \left[1 - \frac{1.062^6 - 1}{1.062^{14} - 1} \right] = 59,053$$

$$L_0 = 88,000.0098$$

The sinking fund payment is:

$$SFP = \frac{L}{s_{\overline{14}|0.062}} = \frac{88,000.0098}{21.3123} = \mathbf{4,129.07}$$

Chapter 12: Project Evaluation

Solution 12.01

B Section 12.01, Net Present Value

The 70,000 received at the end of 3 years earns 3% for 2 years, and the 70,000 received at the end of 4 years earns 3% for 1 year. The net present value is:

$$NPV = \frac{70,000(1.03)^2 + 70,000 \times 1.03}{1.07^5} - 100,000 = \mathbf{4,354.80}$$

Solution 12.02

D Section 12.01, Net Present Value

The net present value of Project A is:

$$NPV_0 = PV_0(\text{Cash Inflows}) - PV_0(\text{Cash Outflows}) = -5,000 + \frac{2,000}{1.08} + \frac{5,000}{1.08^2}$$

$$= 1,138.5460$$

We can set this equal to the next present value of Project B:

$$1,138.5460 = 2,000 + \frac{5,000}{1.08} - \frac{X}{1.08^2}$$

$$X = \mathbf{6,404.80}$$

Solution 12.03

C Section 12.02, Internal Rate of Return

The time-0 equation of value to be solved is:

$$368.15 = 100 + 100v + 200v^2$$

$$0 = 200v^2 + 100v - 268.15$$

We can use the quadratic formula to find v:

$$v = \frac{-100 \pm \sqrt{100^2 + 4 \times 200 \times 268.15}}{2 \times 200}$$

$$v = -1.4346 \quad \text{or} \quad v = 0.9346$$

We discard the negative value of v, and use the positive value to find i:

$$v = 0.9346$$

$$\frac{1}{1+i} = 0.9346$$

$$i = \mathbf{0.07}$$

Alternatively, the following equation, which is a re-ordered version of an equation found above, shows us the values to put into the cash flow worksheet of the BA-II Plus calculator:

$$0 = -268.15 + 100v + 200v^2$$

We can now use the calculator to find the internal rate of return:

[CF] CF0 = 268.15 [+/-] [ENTER] ↓

C01 = 100 [ENTER] ↓ ↓

C02 = 200 [ENTER]

[IRR] [CPT]

The result is 7.00.

The answer is **7.00%**.

Solution 12.04

B Section 12.02, Internal Rate of Return

The time-0 equation of value to be solved is:

$$1,000 + 1,400v = 3,000v^2$$

$$1,000 + 1,400v - 3,000v^2 = 0$$

The easiest way to obtain the annual effective interest rate that satisfies the equation above is to use the BA-II Plus calculator:

[CF] CF0 = 1,000 [ENTER] ↓

C01 = 1,400 [ENTER] ↓ ↓

C02 = 3,000 [+/-] [ENTER]

[IRR] [CPT]

The result is 16.8154%, which is an annual effective interest rate. We must convert it into an interest rate that is convertible semiannually.

[÷] 100 [+] 1 [=] [y^x] 0.5 [–] 1 [=] [×] 2 [=]

Answer is **0.1616**.

Solution 12.05

E Section 12.02, Dollar-Weighted Rate of Return

To find the income we treat the final balance as a withdrawal and we treat the initial balance as a deposit:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = 300 + 600 + 1,400 - 1,000 - 1,200 = 100$$

The exposure of the fund to interest is:

$$\text{Fund exposure} = \sum(\text{Net deposit})(\text{Time deposit is in the fund})$$

$$= 1,000 + 1,200 \times \frac{9}{12} - 300 \times \frac{6}{12} - 600 \times \frac{4}{12} = 1,550$$

The simple interest approximation to the dollar-weighted yield is:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{100}{1,550} = \mathbf{0.0645}$$

Solution 12.06

A Section 12.02, Dollar-weighted Rate of Return

The income is the withdrawals minus the deposits, treating the initial balance as a deposit and the final balance as a withdrawal:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = 115 + 15 + 10 + 120 + 95 - 100 - 20 \times 12$$
$$= 15$$

The fund exposure is the average amount in the fund:

$$\text{Fund exposure} = \sum(\text{Net deposit})(\text{Time deposit is in the fund})$$
$$= 100 + 20\left(\frac{11}{12} + \frac{10}{12} + \cdots + \frac{1}{12} + \frac{0}{12}\right) - 15 \times \frac{10}{12} - 10 \times \frac{6}{12} - 120 \times \frac{3.5}{12} - 95 \times \frac{2}{12}$$
$$= 100 + 20\left(\frac{11 \times 12}{2 \times 12}\right) - \frac{820}{12} = 141.6667$$

The simple interest approximation for the dollar-weighted rate of return is the income divided by the fund exposure:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{15}{141.6667} = \mathbf{0.1059}$$

Solution 12.07

A Section 12.03, Time-Weighted Weight of Return

The time-weighted rate of return is based on the product of the time intervals' corresponding accumulation factors:

$$1 + i = \frac{100,000}{80,000} \times \frac{80,000}{100,000 - 10,000} \times \frac{90,000}{80,000 + 15,000}$$
$$1 + i = \frac{100,000}{80,000} \times \frac{80,000}{90,000} \times \frac{90,000}{95,000}$$
$$i = \mathbf{0.0526}$$

Solution 12.08

D Section 12.03, Time-Weighted Rate of Return

The balance at the end of the year is:

$$10,000 \times 1.065 + 4,200 \times 1.065^{0.5} = 14,984.3512$$

The time-weighted rate of return is:

$$\frac{10,800}{10,000} \times \frac{14,984.3512}{10,800 + 4,200} - 1 = \mathbf{0.0789}$$

Solution 12.09

A Section 12.03, Time-Weighted & Dollar-Weighted Returns

We can use the fact that the time-weighted rate of return is zero to solve for D:

$$(1 + 0) = \frac{20}{16} \times \frac{D}{20 + D}$$
$$\frac{16}{20} = \frac{D}{20 + D}$$
$$320 + 16D = 20D$$
$$D = 80$$

The investment income and the exposure of the fund to interest are:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = D - 16 - D = -16$$
$$\text{Fund exposure} = \sum(\text{Net deposit})(\text{Time deposit is in the fund}) = 16 + 0.5D$$
$$= 16 + 0.5 \times 80 = 56$$

The simple interest approximation for the dollar-weighted rate of return is:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{-16}{56} = -\mathbf{0.2857}$$

Solution 12.10

E Section 12.03, Time-Weighted & Dollar-Weighted Returns

The income and fund exposure are:

$$\begin{aligned}\text{Income} &= \text{Withdrawals} - \text{Deposits} = 110,000 + 20,000 - 100,000 - 15,000 \\ &= 15,000\end{aligned}$$

$$\begin{aligned}\text{Fund exposure} &= \sum(\text{Net deposit})(\text{Time deposit is in the fund}) \\ &= 100,000 + 15,000 \times (1 - t) - 20,000 \times \frac{5}{12} \\ &= 106,666.6667 - 15,000t\end{aligned}$$

The dollar-weighted rate of return is equal to the time-weighted rate of return:

$$\frac{15,000}{106,666.6667 - 15,000t} = \frac{104,000}{100,000} \times \frac{124,000}{119,000} \times \frac{110,000}{104,000} - 1$$

$$\frac{15,000}{106,666.6667 - 15,000t} = 0.1462$$

$$106,666.6667 - 15,000t = 102,586.2069$$

$$t = \mathbf{0.2720}$$

Solution 12.11

C Section 12.03, Time-Weighted & Dollar-Weighted Returns

We can use the simple interest approximation to find the dollar-weighted rate of return for account A. The income and fund exposure are:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = 140 + 8X - 130 - 2X = 10 + 6X$$

$$\begin{aligned}\text{Fund exposure} &= \sum(\text{Net deposit})(\text{Time deposit is in the fund}) \\ &= 130 - 0.75(8X) + 0.5(2X) = 130 - 5X\end{aligned}$$

The dollar-weighted rate of return is approximately:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{10 + 6X}{130 - 5X}$$

We set the dollar-weighted return for Fund A equal to the time-weighted return for Fund B:

$$\frac{10 + 6X}{130 - 5X} = \frac{130}{118} \times \frac{128}{130 - 5X} - 1$$

$$\frac{10 + 6X}{130 - 5X} = \frac{130}{118} \times \frac{128}{130 - 5X} - \frac{130 - 5X}{130 - 5X}$$

$$10 + 6X = \frac{130 \times 128}{118} - 130 + 5X$$

$$X = 1.0169$$

We can substitute the value of X into the expression for the time-weighted return to find the value of i:

$$i = \frac{130}{118} \times \frac{128}{130 - 5X} - 1 = \frac{130}{118} \times \frac{128}{130 - 5 \times 1.0169} - 1 = \mathbf{0.1289}$$

Solution 12.12

C Section 12.04, Defaults

The bank wants to earn a net interest rate of 6%. The stated interest rate on the loans is R_{nom}:

$$e^{R_{net}\times t} = (1-Def)e^{R_{nom}\times t} + Def\,(Rec)e^{R_{nom}\times t}$$
$$e^{0.06\times 5} = (1-0.009)e^{R_{nom}\times 5} + 0.009(0.35)e^{R_{nom}\times 5}$$
$$e^{5R_{nom}} = 1.3578$$
$$R_{nom} = 0.06117$$

The credit spread is the difference between the nominal interest rate and the net interest rate:

$$s = R_{nom} - R_{net} = 0.06117 - 0.06000 = 0.00117$$

Multiplying the credit spread by 10,000, we have the credit spread in basis points:

$$10{,}000s = 10{,}000 \times 0.00117 = \mathbf{11.7}$$

Solution 12.13

A Section 12.04, Defaults

The bank wants to earn a net interest rate of 6%. The stated interest rate on the three-year loans is found below:

$$e^{R_{net}\times t} = (1-Def)e^{R_{nom}\times t} + Def\,(Rec)e^{R_{nom}\times t}$$
$$e^{0.06\times 3} = (1-0.003)e^{R_{nom}\times 3} + 0.003(0.35)e^{R_{nom}\times 3}$$
$$e^{3R_{nom}} = 1.19956$$
$$R_{nom} = 0.06065$$

The credit spread of the three-year loans is the difference between the nominal interest rate and the net interest rate of the three-year loans:

3-year loans: $s = R_{nom} - R_{net} = 0.06065 - 0.06000 = 0.00065$

The stated interest rate on the five-year loans is found below:

$$e^{R_{net}\times t} = (1-Def)e^{R_{nom}\times t} + Def\,(Rec)e^{R_{nom}\times t}$$
$$e^{0.06\times 5} = (1-0.009)e^{R_{nom}\times 5} + 0.009(0.35)e^{R_{nom}\times 5}$$
$$e^{5R_{nom}} = 1.35780$$
$$R_{nom} = 0.06117$$

The credit spread of the five-year loans is the difference between the nominal interest rate and the net interest rate of the five-year loans:

5-year loans: $s = R_{nom} - R_{net} = 0.06117 - 0.06000 = 0.00117$

The difference between the spreads, multiplied by 10,000, is:

$$10{,}000X = 10{,}000(0.00117 - 0.00065) = \mathbf{5.2}$$

Alternatively, without much loss of precision, the credit spread can be estimated by calculating the total default losses as the default rate times the complement of the recovery rate and then annualizing by dividing the total default losses by the term of the loan:

3-year loans: $s \approx \dfrac{0.003 \times (1 - 0.35)}{3} = 0.00065$

5-year loans: $s \approx \dfrac{0.009 \times (1 - 0.35)}{5} = 0.00117$

As before, the difference between the spreads, multiplied by 10,000, is:

$$10,000X = 10,000(0.00117 - 0.00065) = \mathbf{5.2}$$

Solution 12.14

D Section 12.05, Inflation

We are given that the real interest rate is 2.8%:

$$R_r = 2.8\%$$

The actual returns earned each year are shown below:

$$R_a = R_r + I_a$$

First year: $2.8\% + 1.0\% = 3.8\%$

Second year: $2.8\% + 2.0\% = 4.8\%$

Third year: $2.8\% + 4.0\% = 6.8\%$

The payment made at the end of 3 years is:

$$200,000e^{0.038}e^{0.048}e^{0.068} = \mathbf{233,298.18}$$

Alternatively, there is no need to calculate the rate earned each year. The loan earns the real rate of 2.8% for 3 years, and the loan is increased by each year's inflation rate:

$$200,000e^{0.028\times3}e^{0.010}e^{0.020}e^{0.040} = \mathbf{233,298.18}$$

Solution 12.15

D Section 12.05, Inflation

The stated interest rate of a nominal return bond is the nominal interest rate:

$$R_{nom} = R + I_e + I_u + s = 4.0\% + 1.5\% + 0.4\% + 0.8\% = \mathbf{6.7\%}$$

Solution 12.16

B Section 12.05, Inflation

The quoted interest rate of a real return bond is the real interest rate:

$$R_r = R - C + s = 4.0\% - 0.6\% + 0.8\% = \mathbf{4.2\%}$$

Solution 12.17

E Section 12.05, Inflation

The quoted interest rate of a real return loan is the real interest rate:

$$R_r = R - C + s = 4.0\% - 0.6\% + 0.8\% = 4.2\%$$

The actual return that is earned is:

$$R_a = R_r + I_a = 4.2\% + 3.1\% = \mathbf{7.3\%}$$

Solution 12.18

E Section 12.05, Inflation

The quoted interest rate of a real return loan is the real interest rate:

$$R_r = R - C + s = 4.0\% - 0.6\% + 0.8\% = 4.2\%$$

The actual returns earned each year are shown below:

$$R_a = R_r + I_a$$

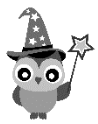
First year: $4.2\% + 1.2\% = 5.4\%$

Second year: $4.2\% + 1.9\% = 6.1\%$

Third year: $4.2\% + 2.8\% = 7.0\%$

The payment made at the end of 3 years is:

$$100,000e^{0.054}e^{0.061}e^{0.070} = \mathbf{120,321.84}$$



Alternatively, there is no need to calculate the rate earned each year. The loan earns the real rate of 4.2% for 3 years, and the loan is increased by each year's inflation rate:

$$100,000e^{0.042\times 3}e^{0.012}e^{0.019}e^{0.028} = \mathbf{120,321.84}$$

Solution 12.19

E Section 12.05, Inflation

The stated interest rate of a nominal return loan is the nominal interest rate:

$$R_{nom} = R + I_e + I_u + s = 4.0\% + 1.5\% + 0.4\% + 0.8\% = 6.7\%$$

At the end of three years, Anita pays:

$$100,000e^{0.067\times 3} = 122,262.48$$

The quoted interest rate of a real return loan is the real interest rate:

$$R_r = R - C + s = 4.0\% - 0.6\% + 0.8\% = 4.2\%$$

The loan earns the real rate of 4.2% for 3 years, and the loan is increased by each year's inflation rate, so Dave pays:

$$100,000e^{0.042\times 3}e^{0.012}e^{0.019}e^{0.028} = 120,321.84$$

The difference between Anita's payment and Dave's payment is:

$$122,262.48 - 120,321.84 = \mathbf{1,940.63}$$

Solution 12.20

B Section 12.05, Inflation

The stated interest rate of a nominal return loan is the nominal interest rate:

$$R_{nom} = R + I_e + I_u + s = 4.0\% + x\% + 0.4\% + 0.8\% = 5.2\% + x\%$$

At the end of three years, Anita pays:

$$100,000e^{(0.052+x\%)\times 3}$$

The quoted interest rate of a real return loan is the real interest rate:

$$R_r = R - C + s = 4.0\% - 0.6\% + 0.8\% = 4.2\%$$

The loan earns the real rate of 4.2% for 3 years, and the loan is increased by each year's inflation rate, so Dave pays:

$$100,000e^{0.042\times 3}e^{0.012}e^{0.019}e^{0.028}$$

Setting Anita's payment equal to Dave's payment allows us to solve for the compensation for expected inflation, x:

$$100,000e^{(0.052+x\%)\times 3} = 100,000e^{0.042\times 3}e^{0.012}e^{0.019}e^{0.028}$$

$$(0.052 + x\%) \times 3 = 0.042 \times 3 + 0.012 + 0.019 + 0.028$$

$$x\% = 0.009667$$

$$x = \mathbf{0.9667}$$

Chapter 13: Fixed Income Securities

Solution 13.01

C Section 13.01, Pricing Noncallable Bonds

The formula for the price of the bond can be used to find the redemption value:

$$P = Coup \times a_{\overline{n}|y} + Rv^n$$

$$129.24 = 3.5 \times a_{\overline{20}|0.02} + \frac{R}{1.02^{20}}$$

$$129.24 = 3.5 \times 16.3514 + \frac{R}{1.02^{20}}$$

$$R = \mathbf{107.00}$$

We can use the BA II Plus to answer this question:

20 [*N*] 2 [*I/Y*] 129.24 [+/-] [*PV*] 3.5 [*PMT*]

[CPT] [*FV*]

Answer is **107.00**.

Solution 13.02

D Section 13.01, Pricing Noncallable Bonds

We can use the BA II Plus to obtain the quarterly effective yield. We multiply by 4 to obtain the nominal yield convertible quarterly:

100 [*N*] 925 [+/-] [*PV*] 1,000 [×] 0.10 [÷] 4 [=] [*PMT*] 1,000 [*FV*]

[CPT] [*I/Y*]

Result is 2.7189.

[×] 4 [=]

Answer is **10.88%**.

Solution 13.03

B Section 13.01, Pricing Noncallable Bonds

The 6-month effective yield is:

$$y = \left(\frac{1,000}{556.84}\right)^{\frac{1}{24}} - 1 = 0.02469$$

The price of the bond is:

$$P = Coup \times a_{\overline{n}|y} + Rv^n = 25 \times a_{\overline{30}|0.02469} + \frac{1,000}{1.02469^{30}}$$

$$= 25 \times \frac{1 - 1.02469^{-30}}{0.02469} + 481.0199 = 25 \times 21.0157 + 481.0199 = \mathbf{1,006.41}$$

The BA-II Plus can be used to answer this question:

1,000 [÷] 556.84 [=] [y^x] 24 [1/*x*] [=] [–] 1 [=] [×] 100 [=] [*I/Y*]

30 [*N*] 25 [*PMT*] 1,000 [*FV*]

[CPT] [*PV*]

Result is –1,006.41. Answer is **1,006.41**.

Solution 13.04

D Section 13.01, Pricing Noncallable Bonds

The price of the bond is:

$$P = Coup \times a_{\overline{n}|y} + Rv^n = 40 \times a_{\overline{60}|0.045} + \frac{1,150}{1.045^{60}} = 40 \times 20.6380 + 81.9824$$

$$= \mathbf{907.50}$$

We can use the BA II Plus to answer this question:

60 [*N*] 4.5 [*I*/*Y*] 40 [*PMT*] 1,150 [*FV*] [CPT] [*PV*]

Result is –907.50. Answer is **907.50**.

Solution 13.05

E Section 13.01, Pricing Noncallable Bonds

The price of the bond is:

$$P = Coup \times a_{\overline{2n}|i} + Rv^{2n} = 1,000r \times \frac{1 - v^{2n}}{i} + 543$$

$$= 1,000 \times 1.25(1 - v^{2n}) + 543 = 1,250(1 - 0.7026^2) + 543$$

$$= \mathbf{1,175.94}$$

Solution 13.06

A Section 13.01, Pricing Noncallable Bonds

The redemption value of the bond is found below:

$$P = Coup \times a_{\overline{2n}|i} + Rv^{2n}$$

$$1,225 = 1,000r \times \frac{1 - v^{2n}}{i} + Rv^{2n}$$

$$1,225 = 1,000 \times 1.25(1 - 0.7026^2) + R \times 0.7026^2$$

$$R = \mathbf{1,199.36}$$

Solution 13.07

C Section 13.01, Pricing Noncallable Bonds

The equation of value that equates the prices of the two bonds is:

$$30 \times a_{\overline{40}|0.03} + \frac{1,100}{1.03^{40}} = 30 \times a_{\overline{40}|i/2} + \frac{912}{\left(1 + \frac{i}{2}\right)^{40}}$$

The easiest way to find i is to use the BA II Plus calculator:

40 [*N*] 3 [*I*/*Y*] 30 [*PMT*] 1,100 [*FV*] [CPT] [*PV*]

Result is –1,030.6557.

912 [*FV*] [CPT] [*I*/*Y*] [×] 2 [=]

Result is 5.4984. Answer is **5.50%**.

Solution 13.08

E Section 13.01, Pricing Noncallable Bonds

The equation of value that equates the prices of the two bonds is:

$$0.03X \times a_{\overline{40}|0.03} + \frac{1.1X}{1.03^{40}} = 0.03X \times a_{\overline{40}|i/2} + \frac{0.895X}{\left(1+\frac{i}{2}\right)^{40}}$$

$$0.03 \times a_{\overline{40}|0.03} + \frac{1.1}{1.03^{40}} = 0.03 \times a_{\overline{40}|i/2} + \frac{0.895}{\left(1+\frac{i}{2}\right)^{40}}$$

The easiest way to find *i* is to use the BA II Plus calculator:

40 [*N*] 3 [*I*/*Y*] 0.03 [*PMT*] 1.1 [*FV*] [CPT] [*PV*]

Result is –1.0307.

0.895 [*FV*] [CPT] [*I*/*Y*] [×] 2 [=]

Result is 2.7250. Answer is **5.45%**.

Solution 13.09

A Section 13.01, Pricing Noncallable Bonds

The equation of value can be solved for *R*:

$$P = Coup \times a_{\overline{n}|y} + Rv^n$$

$$100,000 = R \times \frac{0.0495}{2} \times a_{\overline{40}|1.05^{0.5}-1} + \frac{R}{1.05^{20}}$$

$$100,000 = R\left[0.02475 \times \frac{1-1.05^{-20}}{1.05^{0.5}-1} + \frac{1}{1.05^{20}}\right]$$

$$100,000 = R\left[0.02475 \times 25.2322 + \frac{1}{1.05^{20}}\right]$$

$$R = \mathbf{99,861.61}$$

We can use the BA II Plus to answer this question:

1.05 [y^x] 0.5 [–] 1 [=] [×] 100 [=] [*I*/*Y*]

40 [*N*] 0.0495 [÷] 2 [=] [*PMT*] 1 [*FV*] [CPT] [*PV*]

Result is –1.001386.

[1/*x*] [×] 100,000 [=]

Result is –99,861.5081. Answer is **99,861.61**.

Solution 13.10

B Section 13.01, Noncallable Bonds

The bond price is:

$$P = Coup \times a_{\overline{n}|y} + Rv^n = 35 \times a_{\overline{20}|0.025} + \frac{1,000}{1.025^{20}}$$

$$= 35 \times \frac{1-1.025^{-20}}{0.025} + 610.2709 = 35 \times 15.5892 + 610.2709 = 1,155.8916$$

Since the investor borrows the purchase price of the bond at an annual effective rate of 4%, the amount to be repaid at the end of 10 years is:

$$1,155.8916 \times 1.04^{10} = 1,711.0020$$

The proceeds from the invested coupons and the redemption value of the bond at time 10 years is:

$$Coup \times s_{\overline{n|}} + R = 35s_{\overline{20|}0.015} + 1,000 = 35 \times \frac{1.015^{20} - 1}{0.015} + 1,000$$
$$= 35 \times 23.1237 + 1,000 = 1,809.3283$$

The net cash flow at the end of 10 years is equal to her net gain:

$1,809.3283 - 1,711.0020 = \mathbf{98.33}$

We can use the BA II Plus to answer this question:

20 [*N*] 2.5 [*I/Y*] 35 [*PMT*] 1,000 [*FV*]
[CPT] [*PV*] [×] 1.04 [y^x] 10 [=] [STO] 1
(Result is –1,711.0020)
1.5 [*I/Y*] 0 [*PV*] [CPT] [*FV*] [–] 1,000 [=] [+/-]
(Result is 1,809.3283)
[+] [RCL] 1 [=]
Answer is **98.33**.

Solution 13.11

D Section 13.01, Pricing Noncallable Bonds

The semiannual coupon rate of Bond X is $\frac{(i + 0.02)}{2}$ and the semiannual coupon rate of Bond Y is $\frac{(i - 0.02)}{2}$.

The price of Bond X exceeds the price of Bond Y by 2,816.93:

$$10,000\left(\frac{i}{2} + 0.01\right)a_{\overline{20|}i/2} + \frac{10,000}{\left(1 + \frac{i}{2}\right)^2} - \left[10,000\left(\frac{i}{2} - 0.01\right)a_{\overline{20|}i/2} + \frac{10,000}{\left(1 + \frac{i}{2}\right)^2}\right]$$
$$= 2,816.93$$
$$10,000(0.02)a_{\overline{20|}i/2} = 2,816.93$$

We can use the BA II Plus to answer this question:

2,816.93 [÷] 10,000 [÷] 0.02 [=] [*PV*]
20 [*N*] 1 [+/-] [*PMT*] [CPT] [*I/Y*]
[×] 2 [=]
Result is 7.2000. Answer is **7.20%**.

Solution 13.12

B Section 13.01, Noncallable Bonds

At the end of each month, the net cash flow to Steve is the coupon payment from the bond minus the interest on the loan:

$$10,000 \times \frac{0.10}{12} - 3,000 \times \frac{0.08}{12} = 63.3333$$

At the end of 10 years, Steve receives 10,000 from the bond and pays back the 3,000 loan, giving him a net cash flow of:

$10,000 - 3,000 = 7,000$

Since the cost of entering this position is 7,000, the time-0 equation of value is:

$$7,000 = 63.3333 \times a_{\overline{180}|\frac{i^{(12)}}{12}} + \frac{7,000}{\left(1+\frac{i^{(12)}}{12}\right)^{180}}$$

We can use the BA II Plus to answer this question:

180 [*N*] 7,000 [+/-] [*PV*]

10,000 [×] 0.10 [÷] 12 [–] 3,000 [×] 0.08 [÷] 12 [=] [*PMT*]

7,000 [*FV*] [CPT] [*I/Y*]

Result is 0.9048.

[÷] 100 [+] 1 [=] [y^x] 12 [–] 1 [=]

Answer is **0.1141**.

Solution 13.13

C Section 13.01, Noncallable Bonds

The time-0 equation of value can be used to solve for c:

$$708.11 = \frac{300}{1.05^7} + c\left[\frac{1.03}{\sqrt{1.05}} + \left(\frac{1.03}{\sqrt{1.05}}\right)^2 + \cdots + \left(\frac{1.03}{\sqrt{1.05}}\right)^{14}\right]$$

$$494.9056 = c \times \frac{\frac{1.03}{\sqrt{1.05}} - \left(\frac{1.03}{\sqrt{1.05}}\right)^{15}}{1 - \frac{1.03}{\sqrt{1.05}}}$$

$c = \mathbf{34.00}$

Solution 13.14

E Section 13.01, Noncallable Bonds

Since the yield is equal to the coupon rate, the price of the bond is 1,000. Since the investment in the bond results in a yield of 9%, we have the following equation of value:

$$1,000(1.09)^{12} = 40 s_{\overline{24}|(1+i)^{0.5}-1} + 1,000$$

We can use the BA-II Plus calculator to answer this question:

1,000 [×] 1.09 [y^x] 12 [–] 1,000 [=] [*FV*]

24 [*N*] 40 [+/-] [*PMT*] [CPT] [*I/Y*]

Result is 5.1380.

[÷] 100 [+] 1 [=] [x^2] [–] 1 [=]

Solution is **0.1054**.

Solution 13.15

A Section 13.01, Pricing Noncallable Bonds

The first equation of value below sets the value of the first bond equal to the value of the second bond, and the second equation sets the value of the second bond equal to the value of the third bond:

$$0.0517 \times 1,200 \times a_{\overline{n}|} + 1,200v^n = 0.0648 \times 1,050 \times a_{\overline{n}|} + 1,050v^n$$

$$0.0648 \times 1,050 \times a_{\overline{n}|} + 1,050v^n = 950r \times a_{\overline{n}|} + 950v^n$$

The two equations above can be simplified as follows:

$$150v^n = 6 \times a_{\overline{n}|}$$

$$100v^n = (950r - 68.04) \times a_{\overline{n}|}$$

Dividing the first equation into the second equation allows us to solve for r:

$$\frac{150v^n}{100v^n} = \frac{6 \times a_{\overline{n}|}}{(950r - 68.04) \times a_{\overline{n}|}}$$

$$\frac{150}{100} = \frac{6}{(950r - 68.04)}$$

$$950r - 68.04 = 4$$

$$r = \mathbf{0.0758}$$

Solution 13.16

E Section 13.02, Noncallable Bonds

At the end of 15 years, the accumulated value of the proceeds from the bond is:

$$AV_{15} = 35s_{\overline{30}|0.025} + 1,000 = 35 \times 43.9027 + 1,000 = 2,536.5946$$

The 6-month effective yield, y, equates the present value of the proceeds with the purchase price:

$$1,245 = \frac{2,536.5946}{(1+y)^{30}}$$

$$y = 0.02401$$

The nominal yield convertible semiannually is twice the 6-month effective yield:

$$2y = 2 \times 0.02401 = \mathbf{0.0480}$$

We can use the BA II Plus to answer this question:

30 [*N*] 2.5 [*I*/*Y*] 35 [*PMT*] [CPT] [*FV*]

Result is –1,536.5946.

[–] 1,000 [=] [*FV*] 0 [*PMT*] 1,245 [*PV*] [CPT] [*I*/*Y*]

Result is 2.4007

[×] 2 [=]

Answer is **4.80%**.

Solution 13.17

B Section 13.02, Bond Investment Income

The book value at the end of 7 years is the present value of the bond's cash flow over the remaining 8 years:

$$BV_7 = 0.06 \times 1,000 \times a_{\overline{8}|0.04} + \frac{1,000}{1.04^8} = 60 \times 6.7327 + 730.6902$$

$$= 1,134.6549$$

The interest (which is also known as the investment income) portion of the 8th payment is:

$$InvInc_8 = BV_7 \times y = 1,134.6549 \times 0.04 = \mathbf{45.39}$$

The BA-II Plus can be used to answer this question:

8 [*N*] 4 [*I*/*Y*] 0.06 [×] 1,000 [=] [*PMT*] 1,000 [*FV*]

[CPT] [*PV*]
Result is –1,134.6549.
[×] 0.04 [=]
Result is –45.3862. Answer is **45.39**.

Solution 13.18

A Section 15.02, Book Values

The discount accrued with the 7th coupon is 11:

$$BV_t = BV_{t-1} + DA_t$$
$$BV_7 = BV_6 + DA_7$$
$$DA_7 = BV_7 - BV_6$$
$$DA_7 = 11$$

The discount accrued can also be written as:

$$DA_t = (Ry - Coup)v^{n-t+1}$$

We can now solve for n:

$$11 = (3,000 \times 0.035 - 3,000 \times 0.03)v^{n-7+1}$$
$$0.7333 = 1.035^{6-n}$$
$$6 - n = -9.0158$$
$$n = 15.0158$$

The n that was found above is the number of coupons paid, but the question asks for the number of years until the bond matures. Therefore, we must divide the n above by 2:

$$\frac{15.0158}{2} = \mathbf{7.51}$$

Solution 13.19

D Section 13.02, Book Values

The rate of growth of the accumulation of discount is equal to the yield:

$$\frac{DA_{t+k}}{DA_t} = (1+y)^k$$
$$\frac{841.68}{194.49} = (1+y)^{29-12}$$
$$y = 0.09000$$

We use the discount accumulated in the 12th coupon to find the difference between the yield times the redemption value and the coupon:

$$DA_t = (Ry - Coup)v^{n-t+1}$$
$$194.49 = \frac{Ry - Coup}{1.09000^{30-12+1}}$$
$$Ry - Coup = 999.9998$$

The discount at the time of purchase is:

$$\text{Discount} = (Ry - Coup)a_{\overline{n|}y} = 999.9998 \times a_{\overline{30|}0.09000}$$
$$= 999.9998 \times 10.2737 = \mathbf{10,273.66}$$

Solution 13.20

C Section 13.02, Book Values

We can use the book values provided to find the coupon amount:

$$BV_{t+k} = BV_t(1+y)^k - Coup \times s_{\overline{k}|y}$$

$$BV_{15} = BV_{14}(1.05) - Coup \times s_{\overline{1}|0.05}$$

$$2,223.33 = 2,254.60(1.05) - Coup \times 1$$

$$Coup = 144.00$$

We can now use the prospective formula to find n:

$$BV_t = Coup \times a_{\overline{n-t}|y} + Rv^{n-t}$$

$$BV_{14} = 144 \times a_{\overline{n-14}|0.05} + 2,000v^{n-14}$$

$$2,254.60 = 144 \times \frac{1-1.05^{-(n-14)}}{0.05} + 2,000v^{n-14}$$

$$2,254.60 = 2,880\left(1-v^{n-14}\right) + 2,000v^{n-14}$$

$$625.40 = 880v^{n-14}$$

$$0.7107 = 1.05^{14-n}$$

$$14 - n = -7.0000$$

$$n = \mathbf{21.00}$$

We can use the BA II Plus to answer this question:

2,254.60 [×] 1.05 [–] 2,223.33 [=]

Result is 144.

[*PMT*] 5 [*I/Y*] 2,254.60 [+/-] [*PV*] 2,000 [*FV*]

[CPT] [*N*]

Result is 7.0000.

[+] 14 [=]

Answer is **21.00**.

Solution 13.21

A Section 13.03, Callable Bonds

We can use the BA II Plus to obtain the number of 6-month periods that the bond was held. We then divide by 2 to obtain the number of years that the bond was held:

4.5 [*I/Y*] 988 [+/-] [*PV*] 40 [*PMT*] 1,150 [*FV*]

[CPT] [*N*]

Result is 22.0076.

[÷] 2 [=]

Answer is **11.00**.

Solution 13.22

D Section 13.03, Callable Bonds

Since this bond is a discount bond, its price-to-worst is found by assuming that the bond is held until maturity. Therefore, the highest price that the investor can pay and be assured of the desired yield is 803.64.

The bond is called after 15 years for 1,030, and therefore the time-0 equation of value is:

$$803.64 = 60 \times a_{\overline{15}|} + \frac{1,030}{(1+y)^{15}}$$

The BA-II Plus can be used to find the yield that satisfies the equation of value:

15 [*N*] 803.64 [+/-] [*PV*] 60 [*PMT*] 1,030 [*FV*]

[CPT] [*I*/*Y*]

Result is 8.4663.

Answer is **8.47%**.

Solution 13.23

B Section 13.03, Callable Bonds

The bond is a discount bond, because the coupon is less than the yield-to-worst times the redemption value:

$$0.045 \times 1,000 < 0.05 \times 1,000 \qquad \Rightarrow \qquad Coup < YTW \times R_{t_k}$$

Since the bond is a discount bond, its price-to-worst is calculated based on the latest possible redemption:

$$P = 45 \times a_{\overline{24}|0.05} + 1,000v^{24} = 45 \times 13.7986 + 310.0679 = \mathbf{931.01}$$

We can use the BA II Plus to answer this question:

24 [*N*] 5 [*I*/*Y*] 45 [*PMT*] 1,000 [*FV*] [CPT] [*PV*]

Result is –931.0068. Answer is **931.01**.

Solution 13.24

C Section 13.03, Callable Bonds

The coupon is greater than the product of the yield-to-worst and the final redemption value, so the bond is a premium bond:

$$0.035 \times X > 0.025 \times X \qquad \Rightarrow \qquad Coup > YTW \times R_{t_k}$$

Since the bond is a premium bond, the yield-to-worst can be found by identifying each interval defined by level redemption prices and considering the possibility that the bond is called at the beginning of each interval. In this case, there is only one such interval, and it runs from time 10 years until maturity. The yield-to-worst is based on the beginning of the interval, which occurs at the time 10 years. The equation of value is:

$$1,733.84 = 0.035Xa_{\overline{20}|0.025} + \frac{X}{1.025^{20}}$$

We can use the BA II Plus to answer this question:

20 [*N*] 2.5 [*I*/*Y*] 0.035 [*PMT*] 1 [*FV*]

[CPT] [*PV*]

Result is –1.1559.

[1/*x*] [×] 1,733.84 [=]

Result is –1,500.0022. Answer is **1,500.00**.

Solution 13.25

D Section 13.03, Callable Bonds

We observe that the coupon of 50 is greater than the yield-to-worst times the final redemption value:

$$YTW \times R = 0.03 \times 1,200 = 36$$

Therefore, the bond is a premium bond, and the earliest possible redemption within each interval of level redemption prices should be considered. The price-to-worst is the minimum of the two resulting prices:

$$Min\left[50 \times a_{\overline{20}|0.03} + \frac{1,300}{1.03^{20}},\ 50 \times a_{\overline{30}|0.03} + \frac{1,200}{1.03^{30}}\right]$$

We can use the BA II Plus to answer this question:

20 [*N*] 3 [*I*/*Y*] 50 [*PMT*] 1,300 [*FV*]

[CPT] [*PV*]

Result is –1,463.6522.

[STO] 1

30 [*N*] 1,200 [*FV*] [CPT] [*PV*]

Result is –1,474.4062.

Since 1,463.6522 is less than 1,474.4062, the price-to-worst is **1,463.65**.

Solution 13.26

B Section 13.03, Callable Bonds

We observe that the coupon of 50 is greater than the yield-to-worst times the final redemption value:

$$YTW \times R = 0.03 \times 1,200 = 36$$

Therefore, the bond is a premium bond, and the earliest possible redemption within each interval of level redemption prices should be considered. The price-to-worst is the minimum of the two resulting prices:

$$Min\left[50 \times a_{\overline{20}|0.03} + \frac{1,400}{1.03^{20}},\ 50 \times a_{\overline{30}|0.03} + \frac{1,200}{1.03^{30}}\right]$$

We can use the BA II Plus to answer this question:

20 [*N*] 3 [*I*/*Y*] 50 [*PMT*] 1,400 [*FV*]

[CPT] [*PV*]

Result is –1,519.0198.

[STO] 1

30 [*N*] 1,200 [*FV*] [CPT] [*PV*]

Result is –1,474.4062.

Since 1,474.4062 is less than 1,519.0198, the price-to-worst is **1,474.41**.

Solution 13.27

C Section 13.03, Callable Bonds

The coupon is less than the product of the yield-to-worst and the final redemption value, so the bond is a discount bond:

$$0.015 \times X < 0.02 \times X \quad \Rightarrow \quad Coup < YTW \times R_{t_k}$$

Therefore, the yield-to-worst is based on the latest possible redemption, which occurs at the end of 20 years. The equation of value is:

$$1{,}104.92 = 0.015Xa_{\overline{40}|0.02} + \frac{X}{1.02^{40}}$$

We can use the BA II Plus to answer this question:

40 [*N*] 2 [*I*/*Y*] 0.015 [*PMT*] 1 [*FV*]

[CPT] [*PV*]

Result is –0.8632.

[1/*x*] [×] 1,104.92 [=]

Result is –1,279.9943. Answer is **1,279.99**.

Solution 13.28

E Section 13.03, Callable Bonds

Since the price is less than the redemption value of 1,200, the bond is a discount bond. Therefore, the yield-to-worst is based on the latest possible redemption, which occurs at the end of 15 years. The equation of value is:

$$1{,}150 = (0.025 \times 1{,}200)a_{\overline{30}|y} + \frac{1{,}200}{(1+y)^{30}}$$

We can use the BA II Plus to answer this question:

30 [*N*] 1150 [+/-] [*PV*] 0.025 [×] 1,200 [=] [*PMT*] 1,200 [*FV*]

[CPT] [*I*/*Y*]

Result is 2.7045.

[×] 2 [=]

Answer is **5.4091%**.

Solution 13.29

A Section 13.04, Government Bonds

I is true. Negative TIPS yields indicate a negative real rate of return, and the real rate of return is the compensation for deferred consumption minus the cost of inflation protection.

II is false. Retirees that invest in nominal return U.S. Treasury bonds are not protected from inflation risk.

III is false. A city can go into default if the tax receipts are not sufficient to meet the city’s obligations.

Since only I is true, the correct answer is **Choice A**.

Solution 13.30

B Section 13.04, Government Bonds

I is false. Revenue bonds must use the funds from a particular project to support the bonds. General obligation bonds are backed by the general taxing authority of the issuer.

II is true. In the United States, bonds issued by state and local governments are often tax-exempt or taxable at a preferred rate.

III is false. Canadian municipal bonds are not free of default risk.

IV is false. The government of Canada issues some bonds that are denominated in U.S. dollars.

Since only II is true, the correct answer is **Choice B**.

Solution 13.31

A Section 13.05, Corporate Bonds

Choice A is true. The bid price that a dealer is willing to pay is less than the price that a dealer asks for a bond. Therefore, the bid yield is greater than the ask yield.

Choice B is false. The higher the bid-ask spread of a bond, the more expensive it is to convert the bond into cash. Therefore, the higher the bid-ask spread, the less liquid a bond is.

Choice C is false. A call provision works to the detriment of the owner of the bond. Therefore, the yield must be higher to entice an investor to purchase a callable bond over an otherwise equivalent bond that does not have a call provision.

Choice D is false. A put provision benefits the owner of the bond. Therefore the purchaser will accept a lower yield when comparing a puttable bond to an otherwise equivalent bond that does not have a put provision.

Choice E is false. Although credit risk is often the largest component of the yield spread to Treasuries, other factors such as liquidity, call provisions, and put provisions can also affect the yield spread.

It might be tempting to argue that Choice E could be true, but when answering multiple choice questions, we must select the best answer. Choice A is clearly true, while Choice E is ambiguous. Therefore, Choice A is the best answer.

Solution 13.32

D Section 13.06, U.S. T-bills

The quoted rate is a simple discount rate based on a 360-day year. Therefore, the interest is the quoted rate times the maturity value times the fraction of the 360-day year until maturity:

$$\begin{aligned} Interest &= QR_{US} \times MaturityValue \times \frac{DaysToMaturity}{360} \\ &= 0.045 \times 1{,}000{,}000 \times \frac{95}{360} = \mathbf{11{,}875.00} \end{aligned}$$

Solution 13.33

E Section 13.06, U.S. T-bills

The purchase price is:

$$\begin{aligned} Price &= MaturityValue\left[1 - QR_{US} \times \frac{DaysToMaturity}{360}\right] \\ &= 1{,}000{,}000\left[1 - 0.045 \times \frac{95}{360}\right] = 988{,}125.00 \end{aligned}$$

After 35 days, the remaining number of days until maturity is:

$$DaysToMaturity = 95 - 35 = 60$$

After 35 days, Charlotte sells the T-bill for:

$$\begin{aligned} Price &= MaturityValue\left[1 - QR_{US} \times \frac{DaysToMaturity}{360}\right] \\ &= 1{,}000{,}000\left[1 - 0.036 \times \frac{60}{360}\right] = 994{,}000.00 \end{aligned}$$

The annual effective yield earned over the 35 days is found below:

$$(1+i)^{\frac{35}{365}} = \frac{994{,}000.00}{988{,}125.00}$$

$$i = \mathbf{6.3771\%}$$

Solution 13.34

B Section 13.07, Canadian T-bills

The quoted rate is:

$$QR_C = \frac{365}{DaysToMaturity} \times \frac{Interest}{Price} = \frac{365}{73} \times \frac{100{,}000 - 99{,}304.87}{99{,}304.87} = 3.5000\%$$

The annual effective yield is:

$$j = \left(\frac{100{,}000}{99{,}304.87}\right)^{365/73} - 1 = 3.5493\%$$

The difference is:

$$j - QR_C = 3.5493\% - 3.5000\% = \mathbf{0.04934\%}$$

Solution 13.35

C Section 13.07, Canadian T-bills

The purchase price is:

$$Price = \frac{MaturityValue}{1 + QR_C \times \frac{DaysToMaturity}{365}} = \frac{1{,}000{,}000}{1 + 0.045 \times \frac{95}{365}} = 988{,}423.26$$

After 35 days, the remaining number of days until maturity is:

$$DaysToMaturity = 95 - 35 = 60$$

After 35 days, the T-bill is sold for:

$$Price = \frac{MaturityValue}{1 + QR_C \times \frac{DaysToMaturity}{365}} = \frac{1{,}000{,}000}{1 + 0.036 \times \frac{60}{365}} = 994{,}117.01$$

The annual effective yield earned over the 35 days is found below:

$$(1+i)^{\frac{35}{365}} = \frac{994{,}117.01}{988{,}423.26}$$

$$i = \mathbf{6.1731\%}$$

Solution 13.36

A Section 13.07, T-bills

Let's consider each T-bill's price. We use *A* to represent the price of T-bill A, *B* to represent the price of T-bill B, and so on:

$$A = MaturityValue\left[1 - QR_{US} \times \frac{DaysToMaturity}{360}\right] = 100,000\left[1 - 0.05 \times \frac{180}{360}\right]$$
$$= 97,500.00$$

$$B = \frac{MaturityValue}{1 + QR_C \times \dfrac{DaysToMaturity}{365}} = \frac{100,000}{1 + 0.05 \times \dfrac{180}{365}} = 97,593.58$$

$$C = \frac{MaturityValue}{(1+i)^t} = \frac{100,000}{1.05^{(180/365)}} = 97,622.63$$

$$D = \frac{MaturityValue}{\left(1 + \dfrac{i^{(m)}}{m}\right)^{tm}} = \frac{100,000}{\left(1 + \dfrac{0.05}{365}\right)^{180}} = 97,564.56$$

$$E = MaturityValue \times e^{-rt} = 100,000 \times e^{-0.05 \times 180/365} = 97,564.40$$

Since each of the T-bills matures for 100,000, the T-bill with the highest yield is the T-bill with the lowest price. T-bill A has the lowest price, so **Choice A** is the correct answer. Although there is no need to calculate each of the annual effective yields, the annual effective yields are calculated below for illustration purposes:

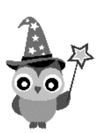
$$i_A = \left(\frac{100,000}{97,500}\right)^{365/180} - 1 = 5.27\%$$

$$i_B = \left(\frac{100,000}{97,593.58}\right)^{365/180} - 1 = 5.06\%$$

$$i_C = \left(\frac{100,000}{97,622.63}\right)^{365/180} - 1 = 5.00\%$$

$$i_D = \left(\frac{100,000}{97,564.56}\right)^{365/180} - 1 = 5.13\%$$

$$i_E = \left(\frac{100,000}{97,564.40}\right)^{365/180} - 1 = 5.13\%$$



Actually, there is no need to calculate any of the values above. Instead, as shown below, we can think the question through logically.

Alternatively, recall that a discount rate is always less than its equivalent interest rate. That is, a discount rate converts to an equivalent interest rate that is higher than the discount rate. In this case, a discount rate of 5% converts to an interest rate that is greater than 5%. Likewise, the fact that a discount rate is always less than its equivalent continuously compounded interest rate implies that a discount rate of 5% converts into an equivalent continuously compounded interest rate that is greater than 5%. Therefore, if all of the 5% rates were converted to one consistent basis (such as the annual effective yields shown above), T-bill A would have the highest yield.

Solution 13.37

E Section 13.07, T-bills

The quoted rates for the two T-bills are:

$$QR_{US} = \frac{360}{DaysToMaturity} \times \frac{Interest}{MaturityValue} = \frac{360}{240} \times \frac{100,000 - 97,000}{100,000} = 4.50\%$$

$$QR_{C} = \frac{365}{DaysToMaturity} \times \frac{Interest}{Price} = \frac{365}{240} \times \frac{100,000 - 97,000}{97,000} = 4.70\%$$

The annual effective interest rates for the two T-bills are the same:

$$i_{US} = \left(\frac{100,000}{97,000}\right)^{365/240} - 1 = 4.74\%$$

$$i_{Can} = \left(\frac{100,000}{97,000}\right)^{365/240} - 1 = 4.74\%$$

Choice A is true, because:

$$QR_{US} = 4.50\%$$

Choice B is true, because:

$$4.70\% > 4.50\%$$

Choice C is true, because:

$$4.74\% = 4.74\%$$

Choice D is true, because:

$$4.74\% > 4.50\%$$

Choice E is false, because:

$$4.74\% > 4.70\%$$

Since Choice E is false, **Choice E** is the correct answer.