Course MFE/3F Practice Exam 4 – Solutions

The chapter references below refer to the chapters of the ActuarialBrew.com Study Manual.

Solution 1

D Chapter 1, Prepaid Forward Price of $1

We don't need the information provided in statement (i) to answer this question.

The usual formula for a prepaid forward price is:

$$F_{0,T}^{P}(S) = S(0)e^{-\delta T}$$

Let's find the prepaid forward price of $1, in euros. From the perspective of a euro-denominated investor, the initial asset price is:

$$\frac{1}{x(0)} = \frac{1}{1.30}$$

From the perspective of a euro-denominated investor, the dividend yield of $1 is 6%.

The prepaid forward price of one dollar, in euros, is therefore:

$$\frac{1}{x(0)}e^{-0.06\times 4} = \frac{1}{1.30}e^{-0.06\times 4} = 0.60510$$

The prepaid forward price of $1,000, in euros, is:

$$0.60510\times 1{,}000 = 605.10$$

Solution 2

B Chapter 15, Itô's Lemma

We have:

$$G(t) = 2e^{Z(t)}$$

The partial derivatives are:

$$G_Z = 2e^{Z} \qquad G_{ZZ} = 2e^{Z} \qquad G_t = 0$$

This results in:

$$\begin{aligned} dG(t) &= G_Z dZ + \frac{1}{2}G_{ZZ}(dZ)^2 + G_t dt = 2e^{Z(t)}dZ(t) + \frac{1}{2}(2)e^{Z(t)}\left[dZ(t)\right]^2 + 0dt \\ &= 2e^{Z(t)}dZ(t) + e^{Z(t)}\left[dZ(t)\right]^2 \\ &= 2e^{Z(t)}dZ(t) + e^{Z(t)}dt \qquad \text{(because } [dZ(t)]^2 = dt\text{)} \\ &= G(t)dZ(t) + 0.5G(t)dt \\ &= 0.5G(t)dt + G(t)dZ(t) \end{aligned}$$

Dividing both sides by $G(t)$ results in:

$$\frac{dG(t)}{G(t)} = 0.5dt + dZ(t)$$

Solution 3

D Chapters 3 and 4, Greeks in the Cox-Ross-Rubinstein Model

The values of u and d are:

$$u = e^{\sigma\sqrt{h}} = e^{0.30\sqrt{1}} = 1.34986$$
$$d = e^{-\sigma\sqrt{h}} = e^{-0.30\sqrt{1}} = 0.74082$$

The risk-neutral probability of an upward movement is:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{(0.09-0.05)(1)} - 0.74082}{1.34986 - 0.74082} = 0.49257$$

The stock price tree and its corresponding tree of option prices are:

Stock			American Put		
		91.1059			0.0000
	67.4929			0.9275	
50.0000		50.0000	7.3550		2.0000
	37.0409			**14.9591**	
		27.4406			24.5594

If the stock price initially moves down, then the resulting put price is $14.9591. This price is in bold type above to indicate that it is optimal to exercise early at this node:

$$52 - 37.0409 = 14.9591$$

The exercise value of 14.9591 is greater than the value of holding the option, which is:

$$e^{-0.09(1)}\left[(0.49257)2.0000 + (1-0.49257)(24.5594)\right] = 12.2900$$

The current value of the American option is:

$$e^{-0.09(1)}\left[(0.49257)(0.9275) + (1-0.49257)(14.9591)\right] = 7.3550$$

In the CRR model, the stock price after one up movement and one down movement is equal to the initial stock price. Therefore, the formula for theta can be simplified:

$$\theta(S,0) = \frac{V_{ud} - V - (Sud - S)\Delta(S,0) - \frac{(Sud-S)^2}{2}\Gamma(S,0)}{2h}$$

$$= \frac{V_{ud} - V - (50-50)\Delta(S,0) - \frac{(50-50)^2}{2}\Gamma(S,0)}{2h}$$

$$= \frac{V_{ud} - V}{2h}$$

The value of theta is:

$$\theta(S,0) = \frac{V_{ud} - V}{2h} = \frac{2.0000 - 7.3550}{2} = -2.6775$$

Solution 4

A Chapter 9, Delta-Gamma Hedging

The gamma of the position to be hedged immediately after the sale of the call options is the quantity of the call options held times the gamma of each call option:

$$-100 \times 0.052 = -5.20$$

We can solve for the quantity, Q, of the put option that must be purchased to bring the hedged portfolio's gamma to zero:

$$-5.2 + 0.035Q = 0.00$$
$$Q = 148.57$$

The delta of the position becomes:

$$-100 \times 0.582 + 148.57 \times (-0.181) = -85.09$$

The quantity of underlying stock that must be purchased, Q_S, is the opposite of the delta of the position being hedged:

$$Q_S = 85.09$$

Therefore, in order to delta-hedge and gamma-hedge the position, we must buy 85.1 units of stock and purchase 148.6 units of the put option.

Solution 5

D Chapter 3, Replication

The end-of-year payoffs of the call and put options in each scenario are shown in the table below. The rightmost column is the payoff resulting from buying the put option and selling the call option.

Scenario	End of Year Price of Stock A	End of Year Price of Stock B	$P_A(125)$ Payoff	$C_B(50)$ Payoff	$P_A(125) - C_B(50)$ Payoff
1	\$300	\$0	0	0	0
2	\$100	\$0	25	0	25
3	\$50	\$150	75	100	–25

We need to determine the cost of replicating the payoffs in the rightmost column above. We can replicate those payoffs by determining the proper amount of Stock A, Stock B, and the risk-free asset to purchase.

Let's define the following variables:

$A =$ Number of shares of Stock A to purchase

$B =$ Number of shares of Stock B to purchase

$C =$ Amount to lend at the risk-free rate

Since Stock A pays a \$20 dividend at time 0.5, each share of Stock A that is purchased provides its holder with the final price of Stock A at time 1 plus the accumulated value of \$20. Since Stock B pays continuously compounded dividends of 8%, each share of stock B purchased at time 0 grows to $e^{0.08}$ shares of Stock B at time 1.

We have 3 equations and 3 unknown variables:

Scenario 1: $\left[300 + 20e^{0.12(0.5)}\right]A + 0B + Ce^{0.12} = 0$

Scenario 2: $\left[100 + 20e^{0.12(0.5)}\right]A + 0B + Ce^{0.12} = 25$

Scenario 3: $\left[50 + 20e^{0.12(0.5)}\right]A + 150e^{0.08}B + Ce^{0.12} = -25$

Subtracting the first equation from the second equation allows us to solve for A:

$$\left[100 + 20e^{0.12(0.5)}\right]A - \left[300 + 20e^{0.12(0.5)}\right]A = 25$$

$$A = \frac{25}{100 + 20e^{0.12(0.5)} - 300 - 20e^{0.12(0.5)}} = -\frac{1}{8}$$

The equation associated with Scenario 1 can now be used to find C:

$$\left[300 + 20e^{0.12(0.5)}\right]A + 0B + Ce^{0.12} = 0$$

$$C = -\left[300 + 20e^{0.12(0.5)}\right]Ae^{-0.12} = \left[300 + 20e^{0.12(0.5)}\right]\frac{1}{8}e^{-0.12} = 35.6139$$

We use the equation associated with Scenario 3 to find B:

$$\left[50+20e^{0.12(0.5)}\right]A+150e^{0.08}B+Ce^{0.12}=-25$$

$$-\left[50+20e^{0.12(0.5)}\right]\frac{1}{8}+150e^{0.08}B+35.6139e^{0.12}=-25$$

$$B=-0.3462$$

The cost now of replicating the payoffs resulting from buying the put and selling the call is equal to the cost of establishing a position consisting of A shares of Stock A, B shares of Stock B, and C lent at the risk-free rate. Since the price of Stock A is \$100 and the price of Stock B is \$50, we have:

$$100A+50B+C=100\times\frac{-1}{8}+50\times(-0.3462)+35.6139=5.8055$$

Solution 6

B Chapter 18, Black-Derman-Toy Model

In each column of rates, each rate is greater than the rate below it by a factor of:

$$e^{2\sigma_i\sqrt{h}}$$

Therefore, the missing rate in the third column is:

$$0.1977e^{-2\sigma_i\sqrt{1}}=0.1977\times\frac{0.1977}{0.2665}=0.1467$$

The missing rate in the fourth column is:

$$0.4353e^{-2\sigma_i\sqrt{1}}=0.4353\times\frac{0.1530}{0.2168}=0.3072$$

We do not need to calculate the missing rate in the fourth column because the value of a year-3 caplet does not depend on the interest rate in the fourth year, but we included it here for completeness.

The tree of short-term rates is:

			43.53%
		26.65%	
	20.77%		30.72%
15.00%		19.77%	
	17.35%		21.68%
		14.67%	
			15.30%

The caplet pays off only if the interest rate at the end of the second year is greater than 16.00%. The payoff table is:

			N/A
		8.4090	
	0.0000		N/A
0.0000		3.1477	
	0.0000		N/A
		0.0000	
			N/A

The payments have been converted to their equivalents payable at the end of 2 years. The calculations are shown below:

$$\frac{100 \times (0.2665 - 0.1600)}{1.2665} = 8.4090$$
$$\frac{100 \times (0.1977 - 0.1600)}{1.1977} = 3.1477$$

The present value of these payments is the value of the 3-year caplet:

$$V_0 = E^*\left[V_T \times \prod_{i=0}^{T-1} \frac{1}{(1+r_i)}\right]$$
$$= 0.5^2 \times \frac{8.4090}{(1.15)(1.2077)} + 0.5^2 \times \frac{3.1477}{(1.15)(1.2077)} + 0.5^2 \times \frac{3.1477}{(1.15)(1.1735)} = 2.66$$

Solution 7

C Chapter 11, Chooser Options

The price of the chooser option can be expressed in terms of a put option and call option:

$$\text{Price of Chooser Option} = P_{Eur}\left(S_0, K, T\right) + e^{-\delta(T-t_1)} C_{Eur}\left(S_0, Ke^{-(r-\delta)(T-t_1)}, t_1\right)$$

The risk-free interest rate and the dividend yield are equal, so the put option has the same strike price as the call option:

$$\text{Price of Chooser Option} = P_{Eur}\left(100, 95, 4\right) + e^{-0.08(4-2)} C_{Eur}\left(100, 95e^{-(0.08-0.08)(4-2)}, 2\right)$$

$$\text{Price of Chooser Option} = P_{Eur}(100, 95, 4) + e^{-0.08(2)} C_{Eur}(100, 95, 2)$$

We can use put-call parity to find the value of the 2-year call option:

$$C_{Eur}(100,95,2) + Ke^{-0.08(2)} = e^{-0.08(2)} S_0 + P_{Eur}(100,95,2)$$
$$C_{Eur}(100,95,2) + 95e^{-0.08(2)} = 100e^{-0.08(2)} + 11.93$$
$$C_{Eur}(100,95,2) = 16.1907$$

We can now solve for the price of the 4-year put option:

$$\text{Price of Chooser Option} = P_{Eur}(100, 95, 4) + e^{-0.08(2)}C_{Eur}(100, 95, 2)$$
$$28.73 = P_{Eur}(100, 95, 4) + 16.1907e^{-0.08(2)}$$
$$P_{Eur}(100, 95, 4) = 14.9332$$

Solution 8

A Chapter 8, Elasticity

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln(S/K) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(75/80) + (0.085 - 0.04 + 0.5 \times 0.28^2) \times 0.75}{0.28\sqrt{0.75}}$$
$$= -0.00573$$
$$d_2 = d_1 - \sigma\sqrt{T} = -0.00573 - 0.28\sqrt{0.75} = -0.24822$$

We have:

$$N(-d_1) = N(0.00573) = 0.50229$$

$$N(-d_2) = N(0.24822) = 0.59802$$

The value of the European put option is:

$$P_{Eur} = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$
$$= 80e^{-0.085(0.75)} \times 0.59802 - 75e^{-0.04(0.75)} \times 0.50229 = 8.3285$$

The delta of the put option is:

$$\Delta_{Put} = -e^{-\delta T}N(-d_1) = -e^{-0.04(0.75)} \times 0.50229 = -0.48745$$

The elasticity of the put option is:

$$\Omega = \frac{S\Delta}{V} = \frac{75 \times (-0.48745)}{8.3285} = -4.3896$$

Solution 9

A Chapter 15, Itô's Lemma

The expression for $G(t)$ is:

$$G = S^t$$

The partial derivatives are:

$$G_S = tS^{t-1}$$
$$G_{SS} = t(t-1)S^{t-2}$$
$$G_t = S^t \ln S$$

From Itô's Lemma and the multiplication rules, we have:

$$\begin{aligned} dG(t) &= G_S dS(t) + \frac{1}{2} G_{SS} [dS(t)]^2 + G_t dt \\ &= tS^{t-1} dS(t) + 0.5t(t-1)S^{t-2} [dS(t)]^2 + S^t \ln S dt \\ &= tS^{t-1} \left[1.5Sdt + SdZ \right] + 0.5t(t-1)S^{t-2} S^2 dt + S^t \ln S dt \\ &= 1.5tS^t dt + tS^t dZ + 0.5t(t-1)S^t dt + S^t \ln S dt \end{aligned}$$

Since $G = S^t$, let's divide both sides by S^t and reorganize the expression:

$$\begin{aligned} \frac{dG(t)}{G(t)} &= 1.5tdt + tdZ + 0.5t(t-1)dt + \ln S dt \\ &= 1.5tdt + 0.5t^2 dt - 0.5tdt + \ln S dt + tdZ \\ &= \left(0.5t^2 + t + \ln S\right) dt + tdZ \end{aligned}$$

We can find an expression for the natural log of S:

$$G = S^t$$
$$\ln G = t \ln S$$
$$\ln S = \frac{\ln G}{t}$$

Substituting this in for the natural log of S, we have:

$$\frac{dG(t)}{G(t)} = \left(0.5t^2 + t + \frac{\ln G(t)}{t}\right) dt + tdZ$$

Solution 10

B Chapter 1, Put-Call Parity

From put-call parity:

$$C_{Eur}(K,T) + Ke^{-rT} = S_0 + P_{Eur}(K,T)$$
$$Ke^{-rT} = S_0 + P_{Eur}(K,T) - C_{Eur}(K,T)$$
$$90e^{-3r} = 75 + 4.11$$
$$e^{-3r} = 0.879$$
$$-3r = \ln(0.879)$$
$$r = 0.04299$$

The continuously compounded risk-free interest rate is 4.30%.

Solution 11

D Chapter 2, Propositions

Dewey is correct, because the prices of the 70-strike and 75-strike puts violate Proposition 2. According to Proposition 2, if the following is violated, then arbitrage is available:

$$P_{Eur}(K_2) - P_{Eur}(K_1) \le (K_2 - K_1)e^{-rt}$$

Proposition 2 is violated since:

$$P_{Eur}(75) - P_{Eur}(70) > (75.00 - 70.00)e^{-0.07\times 1}$$
$$13.50 - 8.75 > (75.00 - 70.00)e^{-0.07\times 1}$$
$$4.75 > 4.66$$

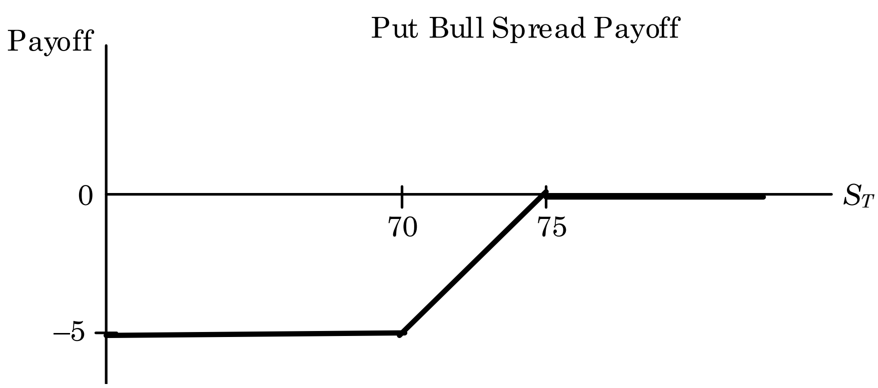
Since Proposition 2 is violated, arbitrage can be obtained by entering into a put bull spread consisting of buying the 70-strike put and selling the 75-strike put. This results in a positive cash flow at time 0 of:

$$P(75) - P(70) = 4.75$$

The positive cash flow can be lent at the risk-free rate, resulting in a positive, time 1 cash flow of:

$$4.75e^{0.07} = 5.0944$$

Any negative cash flows produced by the put bull spread will be more than offset by the $5.0944. The payoffs of the put bull spread are graphed below:



Since $5.0944 is greater than even the worst case result from the put bull spread, Dewey is correct, and his strategy produces arbitrage profits.

Louie is also correct, because the call prices violate Proposition 3. According to Proposition 3, if the following is violated, then arbitrage profits are available:

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \ge \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$

Proposition 3 is violated, since:

$$\frac{C(60)-C(70)}{70-60} < \frac{C(70)-C(75)}{75-70}$$
$$\frac{7-5}{70-60} < \frac{5-3}{75-70}$$
$$0.2 < 0.4$$

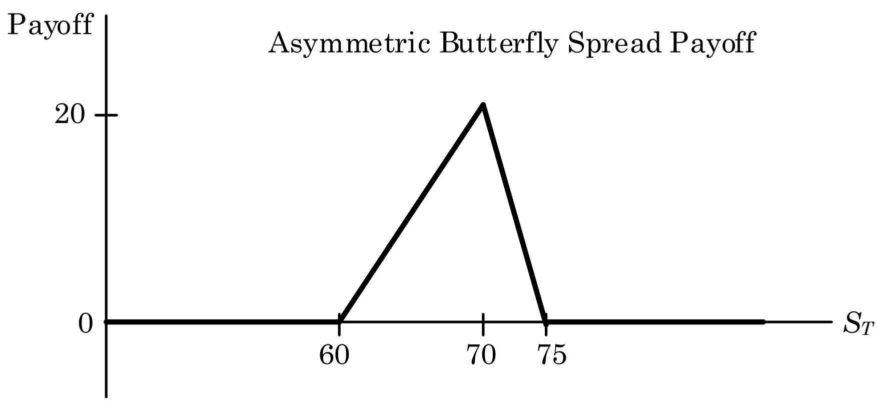
Since Proposition 3 is violated, arbitrage can be obtained by entering into an asymmetric butterfly spread, where λ 60-strike call options are purchased for each 70-strike call option sold and:

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1} = \frac{75-70}{75-60} = \frac{1}{3}$$

Arbitrage is earned if we create an asymmetric butterfly spread by purchasing $\frac{1}{3}$ of a 60-strike call option, selling 1 70-strike call option, and purchasing $\frac{2}{3}$ of a 75-strike call option. We can scale this up by multiplying by 6, giving us the strategy outlined in the question: purchase 2 of the 60-strike calls, sell 6 of the 70-strike calls, and purchase 4 of the 75-strike calls. This results in a positive cash flow at time 0 of:

$$-2\times 7.00 + 6\times 5.00 - 4\times 3.00 = 4.00$$

An asymmetric butterfly spread produces only non-negative payoffs. The payoffs of the asymmetric butterfly spread are graphed below:



Since Louie is being paid to enter into an asymmetric butterfly spread, his strategy produces arbitrage profits.

Solution 12

D Chapter 2, Factors Affecting Premiums

Statement I is false, because if the time to maturity is increased so that it occurs after a liquidating dividend are paid, then the European call option expires worthless. (See page 280 of the third edition of the Derivatives Markets textbook.)

Statement II is true, because before expiration an American call option on a non-dividend paying stock can be sold for more than its exercise value. (See page 277 of the third edition of the Derivatives Markets textbook.)

Statement III is true because an American option can always be turned into a shorter-lived American option. (See page 276 of the third edition of the Derivatives Markets textbook.)

Solution 13

A Chapter 16, Black-Scholes Equation

We can use the Black-Scholes equation to find rV:

$$0.5\sigma^2 S^2 V_{SS} + (r-\delta)SV_S + V_t = rV$$

$$0.5(0.30)^2(5)^2(0.245) + (0.08-0.02)(5)(0.624) - 0.405 = rV$$

$$0.057825 = rV$$

The expected change per unit of time under the risk-neutral distribution is the risk-free rate times the value of the option:

$$\frac{E^*[dV]}{dt} = rV = 0.057825$$

Alternative Solution

The stock process under the risk-neutral probability measure is:

$$dS(t) = (r-\delta)S(t)dt + \sigma S(t)d\tilde{Z}(t)$$

$$dS(t) = (0.08-0.02)S(t)dt + 0.30S(t)d\tilde{Z}(t)$$

$$dS(t) = 0.06S(t)dt + 0.30S(t)d\tilde{Z}(t)$$

Using Itô's Lemma and using the multiplication rules to simplify, we have:

$$\begin{aligned} dV(t) &= V_S dS(t) + 0.5V_{SS}[dS(t)]^2 + V_t dt \\ &= V_S\left[0.06S(t)dt + 0.30S(t)d\tilde{Z}(t)\right] + 0.5V_{SS}\left[0.06S(t)dt + 0.30S(t)d\tilde{Z}(t)\right]^2 + V_t dt \\ &= 0.06V_S S(t)dt + 0.30V_S S(t)d\tilde{Z}(t) + 0.5V_{SS} \times 0.09[S(t)]^2 dt + V_t dt \\ &= \left(0.06V_S S(t) + 0.045V_{SS}[S(t)]^2 + V_t\right)dt + 0.30V_S S(t)d\tilde{Z}(t) \end{aligned}$$

The expected value of $dV(t)$ under the risk-neutral probability measure is:

$$\begin{aligned} E^*[dV(t)] &= E^*\left[\left(0.06V_S S(t) + 0.045V_{SS}[S(t)]^2 + V_t\right)dt + 0.30V_S S(t)d\tilde{Z}(t)\right] \\ &= \left(0.06V_S S(t) + 0.045V_{SS}[S(t)]^2 + V_t\right)dt \qquad \text{because } E^*\left[d\tilde{Z}(t)\right] = 0 \end{aligned}$$

Using the values at time $t = 3$, we have:

$$\frac{E^*[dV(3)]}{dt} = 0.06V_S S(3) + 0.045V_{SS}[S(3)]^2 + V_t$$
$$= 0.06 \times 0.624 \times 5 + 0.045 \times 0.245 \times 5^2 - 0.405$$
$$= 0.057825$$

Solution 14

D Chapter 14, Multiplication Rules

To keep the presentation from becoming too cluttered, we drop the functional relationships and conditional statement from the notation until the end.

We have:

$$dX(t) = 0.05Xdt + 0.40XdZ$$
$$dQ(t) = 0.13Qdt + 0.20QdZ$$

We begin by multiplying the expression out:

$$E\left[dX(t) \times dQ(t) \middle| X(t), Q(t)\right]$$
$$= E[(0.05Xdt + 0.40XdZ) \times (0.13Qdt + 0.20QdZ)]$$
$$= E[0.05Xdt(0.13)Qdt + 0.40X(0.13)QdtdZ + 0.05Xdt(0.20)QdZ + 0.40X(0.20)QdZdZ]$$

The first 3 terms in the expression become zero because of the following multiplication rules:

$$(dt)^2 = 0$$
$$dt \times dZ = 0$$

For the fourth term, we make use of the following multiplication rule:

$$dZ \times dZ = dt$$

After applying the multiplication rules, we have:

$$E[0 + 0 + 0 + 0.40(0.20)dZdZXQ]$$
$$= E[0 + 0 + 0 + 0.40(0.20)(dt)XQ]$$
$$= 0.08XQdt$$

Including the functional relationships, this is written as:

$$0.08X(t)Q(t)dt$$

Solution 15

E Chapter 7, Black-Scholes Formula for Currencies

Brad wants to give up \$62,500 and receive €50,000 at the end of 6 months. This can be accomplished with either of 2 different purchases:

- 50,000 dollar-denominated call options on euros, each with a strike price of \$1.25.
- 62,500 euro-denominated put options on dollars, each with a strike price of €0.80.

Therefore, the correct answer must be either Choice B or Choice E.

Let's find the price of the dollar-denominated call options.

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln(x_0 / K) + (r - r_f + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(1.25/1.25) + (0.07 - 0.04 + 0.5 \times 0.12^2)(0.5)}{0.12\sqrt{0.5}}$$
$$= 0.21920$$
$$d_2 = d_1 - \sigma\sqrt{T} = 0.21920 - 0.12\sqrt{0.5} = 0.13435$$

We have:

$$N(d_1) = N(0.21920) = 0.58675$$

$$N(d_2) = N(0.13435) = 0.55344$$

The value of the call option is:

$$C_{Eur} = x_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$
$$= 1.25e^{-0.04(0.5)} \times 0.58675 - 1.25e^{-0.07(0.5)} \times 0.55344 = 0.050909$$

The value of 50,000 of the dollar-denominated call options is:

$$\$0.050909 \times 50,000 = \$2,545.43$$

Therefore, Choice B is not correct.

The 50,000 dollar-denominated calls with a strike price of \$1.25 (described in Choice B) allow their owner the right to give up \$62,500 to obtain €50,000. Likewise, 62,500 euro-denominated puts with a strike price of €0.80 (described in Choice E) allow their owner the right to give up \$62,500 to obtain €50,000.

Since both option positions have the same payoff, they must have the same current cost:

$$\frac{\$2,545.43}{\$1.25/€} = €2,036.35$$

Therefore, Choice E is correct.

Alternatively, we can use a relationship from Chapter 1 of the ActuarialBrew.com Study Manual to find the cost of one of the euro-denominated put options:

$$C_{\$}(x_t, K, T-t) = x_t K P_{€}\left(\frac{1}{x_t}, \frac{1}{K}, T-t\right)$$

$$0.050909 = 1.25 \times 1.25 \times P_{€}(0.80, 0.80, 0.5)$$

$$P_{€}(0.80, 0.80, 0.5) = 0.032582$$

The value of 62,500 of the put options is therefore:

$$62,500 \times €0.032582 = €2,036.35$$

Solution 16

A Chapter 7, Options on Currencies

Since the call option is euro-denominated, we use the euro as the base currency. This means that the current exchange rate is:

$$x_0 = \frac{1}{135}$$

We also have:

$$r = 0.05$$

$$r_f = 0.015$$

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln(x_0 / K) + (r - r_f + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{1/135}{0.007}\right) + (0.05 - 0.015 + 0.5 \times 0.12^2) \times 0.25}{0.12\sqrt{0.25}}$$

$$= 1.11867$$

$$d_2 = d_1 - \sigma\sqrt{T} = 1.11867 - 0.12\sqrt{0.25} = 1.05867$$

We have:

$$N(d_1) = N(1.11867) = 0.86836$$

$$N(d_2) = N(1.05867) = 0.85512$$

The value of the call option is:

$$C_{Eur} = x_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$= \frac{1}{135} e^{-0.015(0.25)} \times 0.86836 - 0.007 e^{-0.05(0.25)} \times 0.85512 = 0.0004967$$

Since the call option is denominated in euros, the price of the call option is 0.0004967 euros.

Solution 17

C Chapter 13, Estimating Parameters of the Lognormal Distribution

The quickest way to do this problem is to input the data into a calculator.

Using the TI-30XS MultiView, the procedure is:

[data] [data] 4 (to clear the data table)

(enter the data below)

L1	L2	L3
62	61	-----------
61	56	
56	64	
64	59	
59	63	
63	64	

(place cursor in the L3 column) [data] $\rightarrow$ (to highlight FORMULA)

1 [ln] [data] 2 / [data] 1) [enter]

[2nd] [quit] [2nd] [stat] 1

Data: (highlight L3) FRQ: (highlight one) [enter]

2 (to obtain $\bar{x}$)

$\bar{x} \times 12$ [enter] The result is 0.063497397

[2nd] [stat] 3

3 (to obtain Sx)

$Sx \times \sqrt{12}$ [enter] The result is 0.295604642

Using the TI-30X IIS, the procedure is:

[2nd][STAT] (Select 1-VAR) [ENTER]

[DATA]

X1= ln(61/62) [ENTER] ↓↓ (Hit the down arrow twice)

X2= ln(56/61) [ENTER] ↓↓

X3= ln(64/56) [ENTER] ↓↓

X4= ln(59/64) [ENTER] ↓↓

X5= ln(63/59) [ENTER] ↓↓

X6= ln(64/63) [ENTER]

[STATVAR] $\rightarrow$ $\rightarrow$ (Arrow over to Sx)

$\times \sqrt{12}$ [ENTER] (The result is 0.295604642)

[STATVAR] $\rightarrow$ (Arrow over to $\bar{x}$)

$\times$ 12 [ENTER] (The result is 0.063497397)

To exit the statistics mode: [2nd] [EXITSTAT] [ENTER]

Alternatively, using the BA II Plus calculator, the procedure is:

[2nd][DATA] [2nd][CLR WORK]

X01= 61/62 = LN [ENTER] ↓↓ (Hit the down arrow twice)

X02= 56/61 = LN [ENTER] ↓↓

X03= 64/56 = LN [ENTER] ↓↓

X04= 59/64 = LN [ENTER] ↓↓

X05= 63/59 = LN [ENTER] ↓↓

X06= 64/63 = LN [ENTER]

[2nd][STAT] ↓↓↓ $\times \sqrt{12}$ = (The result is 0.29560464)

[2nd][STAT] ↓↓ $\times$ 12 = (The result is 0.06349740)

To exit the statistics mode: [2nd][QUIT]

Therefore, the annualized estimates for the mean and standard deviation of the normal distribution are:

$$\frac{\bar{r}}{h} = \hat{\alpha} - \delta - 0.5\hat{\sigma}^2 = 0.063497$$
$$\hat{\sigma} = 0.295605$$

The estimate for the annualized expected return is:

$$\hat{\alpha} = \frac{\bar{r}}{h} + \delta + 0.5\hat{\sigma}^2 = 0.063497 + 0 + 0.5 \times (0.295605)^2 = 0.107188$$

Solution 18

A Chapter 4, Utility Values and State Prices

Since the probability of the high state is 0.60, the probability of the low state is:

$$1 - p = 1 - 0.60 = 0.40$$

The stock's cash flow in the low state can now be determined:

$$S = Q_u S_u e^{\delta h} + Q_d S_d e^{\delta h}$$
$$S = pU_u S_u e^{\delta h} + (1-p)U_d S_d e^{\delta h}$$
$$19.44 = 0.60 \times 0.58 \times 30e^{0\times 3} + 0.40 \times 1.25 \times C_L e^{0\times 3}$$
$$C_L = 18$$

The value of the derivative is:

$$\begin{aligned} V(0) &= Q_u V_u(3) + Q_d V_d(3) \\ &= pU_u \times \ln(30) + (1-p)U_d \times \ln(18) \\ &= 0.60 \times 0.58 \times \ln(30) + 0.40 \times 1.25 \times \ln(18) \\ &= 2.6288 \end{aligned}$$

Solution 19

E Chapter 18, Black-Derman-Toy Model

The yield volatility for the 3-year bond is:

$$\begin{aligned} \sigma_{1,T} &= 0.5 \times \ln\left[\frac{P(1,T,r_u)^{-1/(T-1)} - 1}{P(1,T,r_d)^{-1/(T-1)} - 1}\right] = 0.5 \times \ln\left[\frac{P(1,3,r_u)^{-1/2} - 1}{P(1,3,r_d)^{-1/2} - 1}\right] \\ &= 0.5 \times \ln\left[\frac{0.7560^{-1/2} - 1}{0.8099^{-1/2} - 1}\right] = 0.1501 \end{aligned}$$

Solution 20

D Chapter 11, All-or-Nothing Options

The trick to answering this question quickly is recognizing that d_1 for Stock A is equal to d_2 for Stock B:

$$\begin{aligned} \text{Stock A: } d_1 &= \frac{\ln(S_t / K) + [r - \delta + 0.5\sigma^2](T-t)}{\sigma\sqrt{T-t}} \\ &= \frac{\ln(75/K) + [0.10 - 0.09 + 0.5(0.2)^2](2)}{0.2\sqrt{2}} \\ &= \frac{\ln(75/K) + [0.03](2)}{0.2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{Stock B: } d_2 &= \frac{\ln(S_t / K) + [r - \delta - 0.5\sigma^2](T-t)}{\sigma\sqrt{T-t}} \\ &= \frac{\ln(75/K) + [0.10 - 0.05 - 0.5(0.2)^2](2)}{0.2\sqrt{2}} \\ &= \frac{\ln(75/K) + [0.03](2)}{0.2\sqrt{2}} \end{aligned}$$

For Stock B, we can use the cash call price to determine $N(d_2)$:

$$\begin{aligned} CashCall(K) &= e^{-r(T-t)} N(d_2) \\ 0.39 &= e^{-0.10(2)} N(d_2) \\ N(d_2) &= 0.47635 \end{aligned}$$

This value of $N(d_2)$ replaces $N(d_1)$ in the formula for the asset put on Stock A:

$$\begin{aligned} AssetPut(K) &= S_t e^{-\delta(T-t)} N(-d_1) \\ &= 75e^{-0.09(2)}[1 - N(d_1)] \\ &= 75e^{-0.09(2)}[1 - 0.47635] \\ &= 32.80 \end{aligned}$$

Solution 21

B Chapter 12, Control Variate Estimate

The payoff of an average strike Asian call option is the final stock price minus the average stock price, when that amount is greater than zero:

Average strike Asian call option payoff = $\text{Max}[S_T - \bar{S}, 0]$

These payoffs are shown in the rightmost columns below:

i	Arithmetic Average	Geometric Average	Final Stock Price	$Y_i e^{0.10}$ Arithmetic Option Payoff	$X_i e^{0.10}$ Geometric Option Payoff
1	45.70	45.40	43.70	0.00	0.00
2	99.60	96.40	133.80	34.20	37.40
3	38.00	37.80	43.80	5.80	6.00
4	37.50	36.90	28.40	0.00	0.00

The values in the two rightmost columns are the payoffs, which means that they are the discounted Monte Carlo prices, Y_i and X_i, times e^{rT}.

The estimate for β is:

$$\beta = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{\left(e^{rT}\right)^2 \sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\left(e^{rT}\right)^2 \sum_{i=1}^{n}(X_i - \bar{X})^2}$$

We get the same estimate for β regardless of whether we use the time 0 prices or the time 1 payoffs (as shown in the rightmost expression above). To save time, we use the time 1 payoffs from the table above.

We perform a regression using the sixth column in the table above as the x-values and the fifth column as the y-values. The resulting slope coefficient is:

$$\beta = 0.912861693$$

During the exam, it is more efficient to let the calculator perform the regression and determine the slope coefficient.

Using the TI-30XS Multiview calculator, we first clear the data by pressing [data] [data] ↓↓↓ until Clear ALL is shown, and then press [enter].

Fill out the table as shown below:

L1	L2	L3
0.00	0.00	-----------
37.40	34.20	
6.00	5.80	
0.00	0.00	

Next, press:

[2nd] [quit] (to exit the table)

[2nd] [stat] 2 (i.e., select 2-Var Stats)

Select L1 for x-data and press [enter].

Select L2 for y-data and press [enter].

Select CALC and then [enter]

Exit the data table by pressing [2nd] [quit].

Obtain access to the statistics by pressing [2nd] [stat] 3

Note the following statistics:

$\bar{x} = 10.85$

$\bar{y} = 10$

$a = 0.912861693$ (This is β)

Alternatively, the TI-30X IIS or the BA II Plus can be used to obtain the values above. To use one of those calculators, follow the steps outlined at the end of this solution and then return to this point to continue the solution.

The Monte Carlo estimate for the arithmetic average strike Asian put is:

$$\bar{A} = \bar{Y} = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} V_i(T) = e^{-0.10\times(1-0)}\left[10\right] = 9.0484$$

The Monte Carlo estimate for the geometric average strike Asian put is:

$$\bar{G} = \bar{X} = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} V_i(T) = e^{-0.10\times(1-0)}\left[10.85\right] = 9.8175$$

The true price for the geometric option is \$4.50, so the control variate price of the arithmetic average option is:

$$A^* = \bar{A} + \beta(G - \bar{G}) = 9.0484 + 0.91286(4.50 - 9.8175) = 4.1942$$

Alternative Calculators

Using the TI-30X IIS, the procedure is:

[2nd][STAT] (Select 2-VAR) [ENTER]

[DATA]

X1= 0.00 ↓ (Hit the down arrow once)

Y1= 0.00 ↓

X2= 37.40 ↓

Y2= 34.20 ↓

X3= 6.00 ↓

Y3= 5.80 ↓

X4= 0.00 ↓

Y4= 0.00 [ENTER]

[STATVAR]

Press the right arrow and note the following statistics:

$\bar{x} = 10.85$

$\bar{y} = 10$

$a = 0.912861693$ (This is β)

To exit the statistics mode: [2nd] [EXITSTAT] [ENTER]

Using the BA II Plus calculator, the procedure is:

[2nd][DATA] [2nd][CLR WORK]

X01= 0.00 [ENTER] ↓ (Hit the down arrow once)

Y01= 0.00 [ENTER] ↓

X02= 37.40 [ENTER] ↓

Y02= 34.20 [ENTER] ↓

X03= 6.00 [ENTER] ↓

Y03= 5.80 [ENTER] ↓

X04= 0.00 [ENTER] ↓

Y04= 0.00 [ENTER]

[2nd][STAT]

Press the down arrow and note the following statistics:

$\bar{x} = 10.85$
$\bar{y} = 10$
$b = 0.91286169$ (This is β)

To exit the statistics mode: [2nd][QUIT]

Solution 22

B Chapters 3 & 18, Risk-Neutral Probability

Let's use $P(0,2)$ to denote the price of a 2-year zero-coupon bond that matures for $1.

We can make use of put-call parity:

$$C(90) + 90 \times P(0,2) = S_0 + P(90)$$
$$C(90) + 90 \times P(0,2) = 80 + P(90)$$
$$C(90) - P(90) = 80 - 90 \times P(0,2)$$

We can use the stock prices to determine the risk-neutral probability that the up state of the world occurs:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{(1+r_0)^h - d}{u-d} = \frac{(1.07)^1 - 40/80}{120/80 - 40/80} = 0.57$$

If the up state occurs, then the zero-coupon bond will have a value of $\frac{1}{1.10}$ at time 1, and if the down state occurs, then the zero-coupon bond will have a value of $\frac{1}{1.051}$ at time 1. The time 0 value is found using the risk-neutral probabilities and the risk-free rate at time 0:

$$P(0,2) = \frac{1}{1.07}\left[0.57 \times \frac{1}{1.10} + 0.43 \times \frac{1}{1.051}\right] = 0.86665$$

We can now use the equation for put-call parity described above to find the solution:

$$C(90) - P(90) = 80 - 90 \times P(0,2) = 80 - 90 \times 0.86665 = 2.00$$

Solution 23

D Chapter 5, Partial Expectation

The formula for the partial expectation is:

$$PE\left[S_T \middle| S_T < K\right] = S_t e^{(\alpha-\delta)(T-t)} N(-\hat{d}_1)$$
$$PE\left[S_3 \middle| S_3 < S_0\right] = S_0 e^{(0.10-0.04)3} N(-\hat{d}_1)$$

The values of $\hat{d}_1$ and $N(-\hat{d}_1)$ are:

$$\hat{d}_1 = \frac{\ln\left(\frac{S_0}{S_0}\right) + (\alpha - \delta + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{0 + (0.10 - 0.04 + 0.5(0.25)^2)(3)}{0.25\sqrt{3}} = 0.63220$$

$$N(-\hat{d}_1) = N(-0.63220) = 0.26363$$

We can use the formula for the partial expectation to solve for the current stock price:

$$PE\left[S_3 \middle| S_3 < S_0\right] = S_0 e^{(0.10-0.04)3} N(-\hat{d}_1)$$

$$27.67 = S_0 e^{(0.10-0.04)3} \times 0.26363$$

$$S_0 = 87.6680$$

Solution 24

E Chapter 11, Rolling Insurance Strategy

The values of d_1 and d_2 for a 3-month put option with a strike price that is 95% of the current stock price are:

$$d_1 = \frac{\ln(S/0.95S) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} = \frac{-\ln(0.95) + (0.15 - 0 + 0.5 \times 0.4^2)(0.25)}{0.4\sqrt{0.25}}$$

$$= 0.54397$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.54397 - 0.4\sqrt{0.25} = 0.34397$$

From the normal distribution table:

$$N(-d_1) = N(-0.54397) = 0.29323$$

$$N(-d_2) = N(-0.34397) = 0.36543$$

The cost of a 3-month put option that has a strike price of 95% of the current stock price is a function of the then-current stock price:

$$\begin{aligned} P_{Eur}(S, 0.95S, 0.25) &= 0.95Se^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1) \\ &= 0.95Se^{-0.25\times0.15}N(-d_2) - Se^{-0.25\times0}N(-d_1) \\ &= S\left[0.95e^{-0.0375}(0.36543) - 0.29323\right] \\ &= 0.04115S \end{aligned}$$

Therefore, the first put option can be purchased with 0.04115 shares of stock now, and the value of the first option is:

$$0.04115S = 0.04115F_{0,0}^{P}(S) = 0.04115S(0) = 0.04115 \times 100 = 4.115$$

The second option can be purchased with 0.04115 shares of stock in 3 months. The current value of a share of stock 3 months from now is its prepaid forward price. The stock does not pay dividends, so the prepaid forward price is equal to the stock price, and the time 0 value of the second option is:

$$0.04115F_{0,0.25}^{P}(S) = 0.04115S(0) = 0.04115 \times 100 = 4.115$$

The third option can be purchased with 0.04115 shares of stock in 6 months. The current value of a share of stock 6 months from now is its prepaid forward price, so the time 0 value of the third option is:

$$0.04115F_{0,0.5}^{P}(S) = 0.04115S(0) = 0.04115 \times 100 = 4.115$$

The fourth option can be purchased with 0.04115 shares of stock in 9 months. The current value of a share of stock 9 months from now is its prepaid forward price, so the time 0 value of the fourth option is:

$$0.04115F_{0,0.75}^{P}(S) = 0.04115S(0) = 0.04115 \times 100 = 4.115$$

Summing the prices of the four options, we have:

$$4.115 + 4.115 + 4.115 + 4.115 = 16.46$$

Solution 25

C Chapter 4, Options on Futures Contracts

Since the risk-neutral probability of an up move is equal to the risk-neutral probability of a down move, both probabilities are 50%:

$$p^* = 1 - p^* \qquad \Rightarrow \qquad p^* = 0.5$$

We are given that the ratio of the factors applicable to the futures price is:

$$\frac{u_F}{d_F} = \frac{3}{2}$$

The formula for the risk-neutral probability of an up move can be used to find u_F and d_F:

$$p^* = \frac{1 - d_F}{u_F - d_F}$$

$$p^* = \frac{\frac{1}{d_F} - \frac{d_F}{d_F}}{\frac{u_F}{d_F} - \frac{d_F}{d_F}}$$

$$\frac{1}{2} = \frac{\frac{1}{d_F} - 1}{\frac{3}{2} - 1}$$

$$d_F = 0.8$$

$$u_F = \frac{3}{2} \times d_F = 1.2$$

The tree of futures prices is therefore:

Futures Prices		
		172.8000
	144.0000	
120.0000		115.2000
	96.0000	
		76.8000

The tree of prices for the European put option is:

European Put		
		0.0000
	7.0391	
18.7301		14.8000
	32.3418	
		53.2000

The price of the European put is:

$$e^{-0.10 \times 0.5}\left[(1/2)7.0391 + (1/2)32.3418\right] = 18.7301$$

The tree of prices for the American put option is:

American Put		
		0.0000
	7.0391	
19.5188		14.8000
	34.0000	
		53.2000

If the futures price moves down to \$96, then early exercise is optimal, as indicated above by the bolding for that node.

The price of the American put is:

$$e^{-0.10\times 0.5}\left[(1/2)7.0391+(1/2)34.0000\right]=19.5188$$

The price of the American put option exceeds the price of the European put option by:

$$19.5188-18.7301=0.7887$$

Solution 26

A Chapter 15, Put-call Parity and S^a

Using the expected rate of price appreciation of 11% and the expected return of 15%, we can determine the dividend yield of the stock:

$$0.11=\alpha-\delta$$
$$0.11=0.15-\delta$$
$$\delta=0.04$$

The price at time 0 of the contingent claim that pays $\left[S(1)\right]^3$ at time 1 is:

$$F_{t,T}^P\left[S(T)^a\right]=e^{-r(T-t)}\left[S(0)\right]^a e^{\left[a(r-\delta)+0.5a(a-1)\sigma^2\right](T-t)}$$

$$=e^{-0.10\times(1-0)}\left[2\right]^3 e^{\left[3(0.10-0.04)+0.5\times 3(3-1)0.30^2\right](1-0)}=8e^{0.35}=11.3525$$

The usual expression for put-call parity is:

$$C+Ke^{-rT}=F_{0,T}^P(S)+P$$

In this case, the underlying asset is S^3 and the strike asset is K^3:

$$C+K^3e^{-rT}=F_{0,T}^P(S^3)+P$$
$$C-P=F_{0,T}^P(S^3)-K^3e^{-rT}$$
$$C-P=11.3525-2.1^3\times e^{-0.10\times 1}$$
$$C-P=2.9728$$

The cost of establishing the position consisting of a long call and a short put is 2.9728.

Solution 27

C Chapter 19, Risk-Neutral Version of the Vasicek Model

The process for the short rate follows the Vasicek Model. The true process can be rewritten in the familiar form:

$$dr=0.06(0.12-r)dt+0.07dZ \qquad \Rightarrow \qquad a=0.06,\ b=0.12,\ \sigma=0.07$$

We can use (iii) to obtain the Sharpe ratio:

$$\tilde{Z}(t) = Z(t) - \phi t$$
$$\tilde{Z}(5) = Z(5) - 5\phi$$
$$-0.06 = 0.24 - 5\phi$$
$$\phi = 0.06$$

The parameter $B(5,10)$ is:

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$
$$B(5,10) = \frac{1 - e^{-a(10-5)}}{a} = \frac{1 - e^{-0.06\times 5}}{0.06} = 4.3197$$

We can use the Sharpe ratio to determine $\alpha(0.09,5,10)$:

$$\phi(r,t) = \frac{\alpha(r,t,T) - r}{q(r,t,T)}$$
$$\phi = \frac{\alpha(r,t,T) - r}{B(t,T)\sigma}$$
$$\alpha(r,t,T) = r + B(t,T)\sigma\phi$$
$$\alpha(0.09,5,10) = 0.09 + 4.3197 \times 0.07 \times 0.06 = 0.1081$$

Solution 28

E Chapter 3, Alternative Binomial Trees

If the option has a positive payoff in both the up and down states, then:

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S(u-d)} = e^{-0.12\times 0.5}\frac{(53 - Su) - (53 - Sd)}{Su - Sd} = e^{-0.06}\frac{Sd - Su}{Su - Sd} = -0.9418$$

But $\Delta = -0.64$, so the option must have a zero payoff in one of the two states. The payoff is clearly positive if the stock price is 36.935, so the payoff must be zero in the other state. For a put option, the positive payoff occurs in the low state, suggesting that $Sd = 36.935$ and $Su > 53$.

We can use Δ to solve for Su:

$$-0.64 = \Delta$$
$$-0.64 = e^{-\delta h}\frac{V_u - V_d}{Su - Sd}$$
$$-0.64 = e^{-0.12\times 0.5}\frac{0 - (53 - 36.935)}{Su - 36.935}$$
$$Su = 60.5748$$

The standard binomial model, the Cox-Ross-Rubinstein model and the Jarrow-Rudd model all have the same ratio of u to d:

$$\frac{u}{d} = e^{2\sigma\sqrt{h}}$$

Therefore, the ratio of Su to Sd can be used to obtain σ :

$$\frac{Su}{Sd} = \frac{u}{d}$$
$$\frac{Su}{Sd} = e^{2\sigma\sqrt{h}}$$
$$\frac{60.5748}{36.935} = e^{2\sigma\sqrt{0.5}}$$
$$\sigma = 0.3498$$

Solution 29

A Chapter 5, Covariance of S_t and S_T

To answer this question, we use the following formulas:

$$E\left[\frac{S_T}{S_t}\right] = e^{(\alpha-\delta)(T-t)}$$
$$Var\left[S_T \middle| S_t\right] = S_t^2 e^{2(\alpha-\delta)(T-t)}\left(e^{\sigma^2(T-t)} - 1\right)$$
$$Cov[S_t, S_T] = E\left[\frac{S_T}{S_t}\right]Var\left[S_t \middle| S_0\right]$$

The variance of X is:

$$Var\left[X \middle| S(0)\right] = Var\left[4\left(2S(1) - S(3)\right)\right] = Var\left[8S(1) - 4S(3)\right]$$
$$= 64Var[S(1)] + 16Var[S(3)] + 2 \times 8 \times (-4)Cov[S(1), S(3)]$$

The variances of $S(1)$ and $S(3)$ are:

$$Var\left[S_1 \middle| S_0\right] = 4^2 e^{2(0.07-0.02)(1-0)}\left(e^{0.22^2(1-0)} - 1\right) = 0.8769$$
$$Var\left[S_3 \middle| S_0\right] = 4^2 e^{2(0.07-0.02)(3-0)}\left(e^{0.22^2(3-0)} - 1\right) = 3.3751$$

The covariance is:

$$Cov[S_1, S_3] = E\left[\frac{S_3}{S_1}\right]Var\left[S_1 \middle| S_0\right] = e^{(0.07-0.02)(3-1)} \times 0.8769 = 0.9691$$

We can now find the variance of X:

$$Var\left[X\middle|S(0)\right] = 64Var[S(1)] + 16Var[S(3)] + 2\times 8\times(-4)Cov[S(1),S(3)]$$
$$= 64\times 0.8769 + 16\times 3.3751 + 2\times 8\times(-4)\times 0.9691$$
$$= 48.0992$$

Solution 30

D Chapter 12, Stratified Sampling Method

The stratified sampling method assigns the first, fifth, and ninth uniform (0, 1) random numbers to the segment (0.00, 0.25), the second, sixth and tenth uniform (0, 1) random variables to the segment (0.25, 0.50), and the third, seventh and eleventh uniform (0, 1) random variables to the segment (0.50, 0.75), the fourth, eighth and the twelfth uniform (0, 1) random variables to the segment (0.75, 1.00).

The lowest simulated normal random variables will come from the segment (0.00, 0.25). The lower of the three values in this segment is the ninth one:

$$Min\left[0.540, 0.302, 0.244\right] = 0.244$$

Therefore, we use 0.244 to find the lowest standard normal random variable and the lowest stock price:

$$u_9 = 0.244$$
$$\hat{u}_9 = V_9 = \frac{0.244 + \left[(9-1)\bmod 4\right]}{4} = \frac{0.244+0}{4} = 0.06100$$
$$Z_6 = N^{-1}(0.06100) = -1.54643$$
$$S(T) = S_0 e^{(\alpha-\delta-0.5\sigma^2)T+\sigma z\sqrt{T}} = 10e^{(0.15-0.03-0.5(0.25)^2)1+0.25(-1.54643)\sqrt{1}} = 7.4241$$

The highest simulated normal random variable will come from the segment (0.75, 1.00). The highest of the three values in this segment is the eighth one:

$$Max\left[0.450, 0.909, 0.501\right] = 0.909$$

Therefore, we use 0.909 to find the highest standard normal random variable and the highest stock price:

$$u_8 = 0.909$$
$$\hat{u}_8 = V_8 = \frac{0.909 + \left[(8-1)\bmod 4\right]}{4} = \frac{0.909+3}{4} = 0.97725$$
$$Z_8 = N^{-1}(0.97725) = 2.00000$$
$$S(T) = S_0 e^{(\alpha-\delta-0.5\sigma^2)T+\sigma z\sqrt{T}} = 10e^{(0.15-0.03-0.5(0.25)^2)1+0.25(2.00000)\sqrt{1}} = 18.0173$$

The difference between the largest and smallest simulated stock prices is:

$$18.0173 - 7.4241 = 10.5932$$