Chapter 16: Term Structure of Interest Rates

Solution 16.01

C Section 16.02, Spot Rates

The value of the bond is:

$$\frac{50}{1.05}+\frac{50}{1.06^2}+\frac{1,050}{1.07^3}=\mathbf{949.23}$$

Solution 16.02

E Section 16.02, Spot Rates

We can use the BA-II Plus calculator to answer this question:

50 [÷] 1.05 [+] 50 [÷] 1.06 [x^2] [+] 1,050 [÷] 1.07 [y^x] 3 [=]

(Result is 949.2316)

[+/-] [PV] 3 [N] 50 [PMT] 1,000 [FV]

[CPT] [I/Y]

(Result is 6.9321)

Answer is **6.93%**.

Solution 16.03

C Section 16.02, Spot Rates

Lending annually at 3.0% for 6 years is clearly an inferior strategy, because 3.0% is the lowest possible rate. Likewise lending annually for 3 years at 3.0% and for one 3-year period at 4.0% is clearly inferior to lending for 6 years (two 3-year periods) at 4.0%.

The remaining viable strategies that must be compared are:

Lend for 6 years (two 3-year periods) at 4.0%. This results in an accumulation factor of:

$$\left(1+\frac{0.04}{4}\right)^{24}=1.2697$$

Lend for 1 year at 3.0% and 5 years at 4.4%. This results in an accumulation factor of:

$$\left(1+\frac{0.03}{4}\right)^{4}\left(1+\frac{0.044}{4}\right)^{5\times 4}=1.2823$$

Since the second strategy has a higher accumulation factor, it produces the maximum annual effective rate, which is:

$$1.2823^{1/6}-1=\mathbf{4.23\%}$$

Solution 16.04

B Section 16.02, Spot Rates

The price of a zero-coupon bond as a percentage of its redemption value is equal to the inverse of the accumulation factor achieved by investing in the bond.

Therefore investing X in the 5-month bond, for example, results in an accumulated value of:

$$\frac{X}{0.95}$$

Investing X in each of the bonds results in an accumulated value of:

$$\frac{X}{0.95}+\frac{X}{0.96}+\frac{X}{0.97}+\frac{X}{0.98}+\frac{X}{0.99}$$
$$=X\left[\frac{1}{0.95}+\frac{1}{0.96}+\frac{1}{0.97}+\frac{1}{0.98}+\frac{1}{0.99}\right]=5.1557X$$

Setting this accumulated value equal to 10,000 allows us to solve for X:

$$5.1557X=10,000$$
$$X=\mathbf{1,939.59}$$

Solution 16.05

B Section 16.03, Forward Rates

The spot rates can be used to calculate the forward rate:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$
$$1+f_1=\frac{(1+s_2)^2}{1+s_1}$$
$$1+f_1=\frac{1.085^2}{1.075}$$
$$f_1=\mathbf{9.51\%}$$

Solution 16.06

D Section 16.03, Forward Rates

We are being asked for f_2:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$
$$1+f_2=\frac{(1+s_3)^3}{(1+s_2)^2}$$
$$1+f_2=\frac{\frac{1}{0.85}}{\frac{1}{0.92}}$$
$$f_2=\mathbf{8.24\%}$$

Solution 16.07

E Section 16.03, Forward Rates

The question is asking for the rate that applies from time 5 to time 6.

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_5=\frac{1.095^6}{1.09^5}$$

$$f_5=\mathbf{12.03\%}$$

Solution 16.08

E Section 16.03, Forward Rates

The loan will be made at the interest rate that can be locked in to apply from time 3 to time 4. This is the 1-year forward rate, deferred for 3 years:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_3=\frac{1.085^4}{1.08^3}$$

$$f_3=0.1001$$

The accumulated value of the loan at time 4 is:

$$1,000\times 1.1001=\mathbf{1,100.14}$$

Solution 16.09

D Section 16.03, Forward Rates

The loan will be accumulated from time 3 to time 4 and from time 4 to time 5.

The 1-year forward rate, deferred for 3 years is found below:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_3=\frac{1.085^4}{1.08^3}$$

$$f_3=0.1001$$

The 1-year forward rate, deferred for 4 years is found below:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_4=\frac{1.090^5}{1.085^4}$$

$$f_4=0.1102$$

The accumulated value of the loan at time 5 is:

$$1,000\times 1.1001\times 1.1102=\mathbf{1,221.41}$$

Solution 16.10

B Section 16.03, Forward Rates

The 3-year accumulation factor calculated with the spot rate is the same as the 3-year accumulation factor calculated with the forward rates:

$$(1+s_t)^t = (1+f_0)(1+f_1)\cdots(1+f_{t-1})$$

$$(1+s_3)^3 = (1+f_0)(1+f_1)(1+f_2)$$

$$(1+s_3)^3 = 1.05\times 1.03\times 1.02$$

$$s_3 = \mathbf{3.33\%}$$

Solution 16.11

A Section 16.03, Yield, Spot, and Forward Rates

I. True. When the price of a bond is equal to its par value, there is no premium or discount to compensate for the coupon rate being greater than or less than the yield. Therefore, the coupon is equal to the yield.

II. False. If the spot rates used to discount the cash flows occurring before time n are greater than the n-year spot rate, then the yield on the n-year bond will be greater than the n-year spot rate.

III. False. If the $(n-1)$-year spot rate is greater than the 1-year forward rate deferred for $(n-1)$ years, then the n-year spot rate will be greater than the 1-year forward rate deferred for $(n-1)$ years.

Solution 16.12

C Section 16.03, Forward Rates

The forward rate j applies from time 4 to time 5:

$$j = f_{4,1} = f_4 = \frac{(1+s_5)^5}{(1+s_4)^4} - 1 = \frac{(1+0.02-0.001\times 5+0.001\times 25)^5}{(1+0.02-0.001\times 4+0.001\times 16)^4} - 1$$

$$= \frac{1.040^5}{1.032^4} - 1 = \mathbf{7.26\%}$$

Solution 16.13

C Section 16.02, Spot Rates

The payments occur at times 2, 3, and 4. The present value at time 0 of the annuity is:

$$PV_0 = \frac{10,000}{1.035^2} + \frac{10,000}{1.039^3} + \frac{10,000}{1.044^4} = 26,668.5525$$

The present value at time 0 can be accumulated for one year using the one-year spot rate to find the current value of the annuity in one year:

$$CV_1 = 26,668.5525\times 1.03 = \mathbf{27,468.61}$$

Solution 16.14

A Section 16.03, Forward Rates

Bond B must produce a cash flow at time 2 that is sufficient to grow to 5,000 at a 5.5% interest rate:

$$\frac{5,000}{1.055} = 4,739.3365$$

The combined prices of Bond A and Bond B are:

$$\frac{3,000}{1.04}+\frac{4,739.3365}{1.05^2}=\mathbf{7,183.33}$$

Solution 16.15

D Section 16.05, Yield Curve Theories

An increase in the number of short-term lenders leads to a lower short-term interest rate. Since a downward-sloping yield curve has a relatively high short-term interest rate, Choice A is not correct.

A decrease in the number of short-term borrowers leads to a lower short-term interest rate. Since a downward-sloping yield curve has a relatively high short-term interest rate, Choice B is not correct.

A decrease in the number of long-term lenders leads to a higher long-term interest rate. Since a downward-sloping yield curve has a relatively low long-term interest rate, Choice C is not correct.

A decrease in the number of long-term borrowers leads to a lower long-term interest rate. Since a downward-sloping yield curve has a relatively low long-term interest rate, **Choice D** is the correct answer.

The liquidity preference theory asserts that the yield curve should be upward sloping because investors require higher yields in exchange for accepting the risk of investing for longer periods of time, so Choice E is not correct.

Chapter 17: Interest Rate Swaps

Solution 17.01

D Section 17.01, Establishing the Swap Rate

The swap rate is equal to the coupon rate of a bond that is priced at par. The formula for the coupon rate of a par bond is derived below:

$$F = c \times F \times a_{\overline{n}|spots} + Fv_{spot}^{n}$$

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 1.07^{-5}}{\frac{1}{1.03} + \frac{1}{1.04^2} + \frac{1}{1.05^3} + \frac{1}{1.06^4} + \frac{1}{1.07^5}} = \frac{0.2870}{4.2643} = \mathbf{0.0673}$$

Solution 17.02

A Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

We could use the forward rates to calculate the spot rates, but since the present values calculated with spot rates will be equal to the present values calculated with the corresponding forward rates, we do not need to calculate the spot rates. Instead, we can use the forward rates to find the present values in the formula above. The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - \frac{1}{1.02 \times 1.03 \times 1.035}}{\frac{1}{1.02} + \frac{1}{1.02 \times 1.03} + \frac{1}{1.02 \times 1.03 \times 1.035}} = \frac{0.0804}{2.8519} = \mathbf{0.0282}$$

Solution 17.03

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

We could use the prices of the zero-coupon bonds to calculate the spot rates, but since the present values calculated with spot rates will be equal to the present values calculated with the prices of the zero-coupon bonds, we do not need to calculate the spot rates. Instead, we can use the prices of the zero-coupon bonds to find the present values in the formula above.

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 0.93}{0.98 + 0.96 + 0.93} = \frac{0.07}{2.87} = \mathbf{0.0244}$$

Solution 17.04

C Section 17.01, Establishing the Swap Rate

Since the swap rate is expressed as a quarterly effective interest rate, let's use one quarter as our unit of time.

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - v_{spot}^{4}}{v_{spot}^{1} + v_{spot}^{2} + v_{spot}^{3} + v_{spot}^{4}} = \frac{1 - 0.94}{0.99 + 0.97 + 0.95 + 0.94} = \frac{0.06}{3.85}$$
$$= \mathbf{0.0156}$$

Solution 17.05

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

The price of the zero-coupon bond can be used to find the value of v_{spot}^{8}:

$$58{,}200.91 = 100{,}000 v_{spot}^{8}$$
$$v_{spot}^{8} = 0.5820091$$

The price of the 3% bond can be used to find the present value of the annuity immediate:

$$76{,}777.78 = 3{,}000 \times a_{\overline{8}|spots} + 100{,}000 v_{spot}^{8}$$
$$76{,}777.78 = 3{,}000 \times a_{\overline{8}|spots} + 58{,}200.91$$
$$a_{\overline{8}|spots} = 6.1923$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 0.5820091}{6.1923} = \mathbf{0.0675}$$

Solution 17.06

B Section 17.01, Establishing the Swap Rate

The formula for the swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}}$$

The yield of the zero-coupon bond can be used to find the value of v_{spot}^{8}:

$$v_{spot}^{8} = \frac{1}{1.07^{8}} = 0.5820$$

The yield of the 3% bond can be used to find the price of the bond:

$$P = 3{,}000 \times a_{\overline{8}|6.87\%} + 100{,}000 v^{8} = 3{,}000 \times \frac{1 - 1.0687^{-8}}{0.0687} + \frac{100{,}000}{1.0687^{8}}$$
$$= 76{,}774.2017$$

The price of the 3% bond can be used to find the present value of the annuity immediate:

$$76{,}774.2017 = 3{,}000 \times a_{\overline{8}|spots} + 100{,}000 v_{spot}^{8}$$

$$76{,}774.2017 = 3{,}000 \times a_{\overline{8}|spots} + 58{,}200.91$$

$$a_{\overline{8}|spots} = 6.1911$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 0.5820091}{6.1911} = \mathbf{0.0675}$$

Solution 17.07

C Section 17.01, Establishing the Swap Rate

We begin by finding the 1-year spot rate:

$$s_1 = \frac{103{,}000}{99{,}516.91} - 1 = 0.0350$$

The 1-year spot rate can be used to find the 2-year spot rate:

$$94{,}802.83 = \frac{2{,}000}{1.0350} + \frac{102{,}000}{(1 + s_2)^2}$$

$$s_2 = 0.0480$$

The 1-year and 2-year spot rates can be used to find the 3-year spot rate:

$$129{,}505.11 = \frac{15{,}000}{1.0350} + \frac{15{,}000}{(1.0480)^2} + \frac{115{,}000}{(1 + s_3)^3}$$

$$s_3 = 0.0430$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - 1.0430^{-3}}{\frac{1}{1.0350} + \frac{1}{1.0480^2} + \frac{1}{1.0430^3}} = \frac{0.1187}{2.7580} = \mathbf{0.0430}$$

Alternatively, we can save a little time by finding the present value factors instead of the spot rates:

$$v_{spot}^{1} = \frac{99{,}516.91}{103{,}000} = 0.9662$$

$$94{,}802.83 = 2{,}000 \times 0.9662 + 102{,}000 v_{spot}^{2} \quad \Rightarrow \quad v_{spot}^{2} = 0.9105$$

$$129{,}505.11 = 15{,}000 \times 0.9662 + 15{,}000 \times 0.9105 + 115{,}000 v_{spot}^{3}$$

$$\Rightarrow \quad v_{spot}^{3} = 0.8813$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n}|spots}} = \frac{1 - v_{spot}^{3}}{v_{spot}^{1} + v_{spot}^{2} + v_{spot}^{3}} = \frac{1 - 0.8813}{0.9662 + 0.9105 + 0.8813} = \frac{0.1187}{2.7580}$$

$$= \mathbf{0.0430}$$

Solution 17.08

E Section 17.01, Establishing the Swap Rate

The swap rate is equal to the coupon rate of a bond that is priced at par. The formula for the coupon rate of a par bond is derived below:

$$F = c \times F \times a_{\overline{n|}spots} + Fv_{spot}^{n}$$

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}}$$

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = \mathbf{0.0444}$$

Solution 17.09

E Section 17.01, Interest Rate Swap Payment

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = 0.0444497$$

Since the swap rate is greater than the floating rate, the fixed-rate payer makes a payment to the floating-rate payer:

$$\begin{aligned}\text{Net Payment to floating-rate payer} &= (0.0444497 - 0.04) \times 100{,}000 \\ &= \mathbf{444.97}\end{aligned}$$

Solution 17.10

A Section 17.02, Valuing Interest Rate Swaps

The swap rate is:

$$c = \frac{1 - v_{spot}^{n}}{a_{\overline{n|}spots}} = \frac{1 - 1.045^{-4}}{\frac{1}{1.03} + \frac{1}{1.035^2} + \frac{1}{1.04^3} + \frac{1}{1.045^4}} = \frac{0.1614}{3.6319} = 0.0444497$$

The coupon of the hypothetical fixed-rate bond is the swap rate times the notional amount of the swap:

$$c \times F = 0.0444497 \times 100{,}000 = 4{,}444.97$$

At the end of one year, the value of the swap to the fixed rate payer is the value of the hypothetical floating-rate bond minus the value of the hypothetical fixed-rate bond:

$$\text{Value to fixed-rate payer} = F - PV_t(\text{Fixed-rate bond})$$

$$= 100{,}000 - \left(\frac{4{,}444.97}{1.04} + \frac{4{,}444.97}{1.05^2} + \frac{104{,}444.97}{1.06^3}\right) = \mathbf{4{,}000.27}$$

Solution 17.11

D Section 17.02, Valuing Interest Rate Swaps

Company A receives a fixed interest rate and pays a floating interest rate.

The swap rate is 5.5%, and the coupon of the hypothetical fixed-rate bond is the swap rate times the notional amount of the swap:

$$c \times F = 0.055 \times 100{,}000 = 5{,}500$$

At the end of two years, the value of the swap to the floating rate payer is the value of the hypothetical fixed-rate bond minus the value of the hypothetical floating-rate bond:

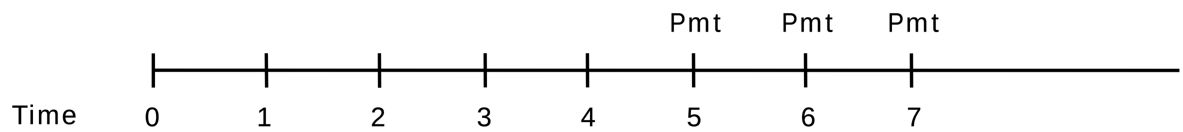
$$\text{Value to floating-rate payer} = PV_t(\text{Fixed-rate bond}) - F$$

$$= \left(\frac{5,500}{1.045} + \frac{5,500}{1.050^2} + \frac{105,500}{1.055^3}\right) - 100,000 = \mathbf{97.0616}$$

Solution 17.12

D Section 17.04, Deferred Interest Rate Swaps

If the swap was not a deferred swap, then there would be 7 payments, which would occur at times 1, 2, 3, 4, 5, 6, and 7. In this case, the payments are made only at times 5, 6, and 7, so the first four payments are not made. That is, the swap is deferred for 4 years:



The swap is deferred for 4 years and matures in 7 years, so we have:

$k = 4$ and $n = 7$

The swap rate is:

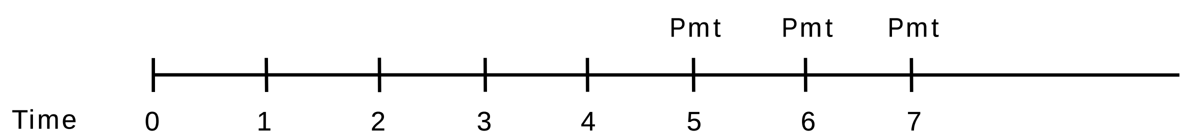
$$c = \frac{P_k - P_n}{P_{k+1} + P_{k+2} + \cdots + P_n} = \frac{P_4 - P_7}{P_5 + P_6 + P_7} = \frac{\dfrac{1}{1.05^4} - \dfrac{1}{1.065^7}}{\dfrac{1}{1.055^5} + \dfrac{1}{1.060^6} + \dfrac{1}{1.065^7}} = \frac{0.17920}{2.11360}$$

$$= \mathbf{0.08478}$$

Solution 17.13

E Section 17.04, Deferred Interest Rate Swaps

The payments are scheduled to be made at times 5, 6, and 7:



At the outset, the swap was a 7-year swap with the payments deferred for 4 years:

$k = 4$ and $n = 7$

The swap rate is:

$$c = \frac{P_k - P_n}{P_{k+1} + P_{k+2} + \cdots + P_n} = \frac{P_4 - P_7}{P_5 + P_6 + P_7} = \frac{\dfrac{1}{1.05^4} - \dfrac{1}{1.065^7}}{\dfrac{1}{1.055^5} + \dfrac{1}{1.060^6} + \dfrac{1}{1.065^7}} = \frac{0.17920}{2.11360}$$

$$= 0.08478$$

The swap was originally scheduled to make payments at times 5, 6, and 7. The current time is time 5, so the second payment will be made in one year.

The next payment is the notional amount times the excess of the fixed rate over the current 1-year spot rate:

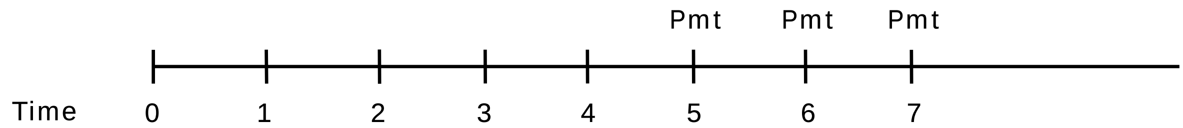
$$\text{Net Payment to floating-rate payer} = (0.08478 - 0.02) \times 100,000$$

$$= \mathbf{6,478.24}$$

Solution 17.14

B Section 17.04, Deferred Interest Rate Swaps

The payments are scheduled to be made at times 5, 6, and 7:



At the outset, the swap was a 7-year swap with the payments deferred for 4 years:

$k = 4$ and $n = 7$

The swap rate is:

$$c = \frac{P_k - P_n}{P_{k+1} + P_{k+2} + \cdots + P_n} = \frac{P_4 - P_7}{P_5 + P_6 + P_7} = \frac{\frac{1}{1.05^4} - \frac{1}{1.065^7}}{\frac{1}{1.055^5} + \frac{1}{1.060^6} + \frac{1}{1.065^7}} = \frac{0.17920}{2.11360}$$

$$= 0.08478$$

The swap was originally scheduled to make payments at times 5, 6, and 7. The current time is time 5, so the second payment will be made in one year.

The current value of the hypothetical floating-rate bond is 100,000.

The current value of the hypothetical fixed-rate bond is based on the swap rate and the current spot rates:

$$PV_5(\text{Fixed-rate bond}) = \frac{8{,}478}{1.02} + \frac{108{,}478}{1.03^2} = 110{,}563.15$$

The value to the fixed-rate payer is:

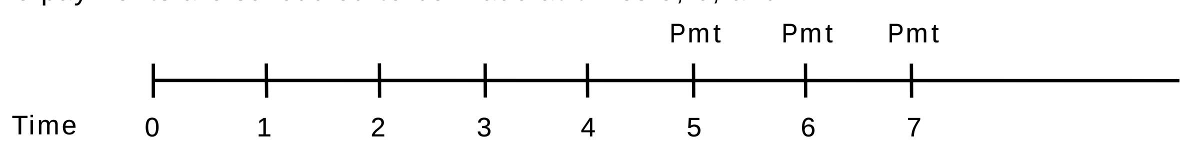
$$\text{Value to fixed-rate payer} = F - PV_5(\text{Fixed-rate bond})$$

$$= 100{,}000 - 110{,}563.15 = \mathbf{-10{,}563.15}$$

Solution 17.15

C Section 17.04, Deferred Interest Rate Swaps

The payments are scheduled to be made at times 5, 6, and 7:



At the outset, the swap was a 7-year swap with the payments deferred for 4 years:

$k = 4$ and $n = 7$

The swap rate is:

$$c = \frac{P_k - P_n}{P_{k+1} + P_{k+2} + \cdots + P_n} = \frac{P_4 - P_7}{P_5 + P_6 + P_7} = \frac{\frac{1}{1.05^4} - \frac{1}{1.065^7}}{\frac{1}{1.055^5} + \frac{1}{1.060^6} + \frac{1}{1.065^7}} = \frac{0.17920}{2.11360}$$

$$= 0.08478$$

The swap was originally scheduled to make payments at times 5, 6, and 7. The current time is time 1, so the first payment will be made in 4 years.

The value of the hypothetical floating-rate bond will be 100,000 at time 4. Now, at time 1, the value of the hypothetical floating-rate bond is found by discounting for 3 years at the current 3-year spot rate:

$$PV_1(\text{Floating-rate bond}) = \frac{100,000}{1.035^3} = 90,194.2706$$

The hypothetical fixed-rate bond makes payments at times 5, 6, and 7. Now, at time 1, those payments occur 4, 5, and 6 years from now, so we use the current 4-year, 5-year, and 6-year spot rates to find the value of the hypothetical fixed-rate bond is:

$$PV_1(\text{Fixed-rate bond}) = \frac{8,478}{1.04^4} + \frac{8,478}{1.055^5} + \frac{108,478}{1.058^6} = 91,078.5926$$

The value to the fixed-rate payer is the value of the floating-rate bond minus the value of the fixed-rate bond:

$$\text{Value to fixed-rate payer} = PV_1(\text{Floating-rate bond}) - PV_1(\text{Fixed-rate bond})$$
$$= 90,194.2706 - 91,078.5926 = -\mathbf{884.3220}$$

Solution 17.16

A Section 17.05, Amortizing Swap

We can use the spot rates to determine the forward rates:

$$f_0 = 0.025$$

$$f_1 = \frac{(1+s_2)^2}{1+s_1} - 1 = \frac{1.0300^2}{1.0250} - 1 = 0.03502$$

$$f_2 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 = \frac{1.0419^2}{1.0300^2} - 1 = 0.06611$$

The swap rate is the weighted average of the forward rates, where the weights are the present values of the notional amounts:

$$c = \frac{\sum_{j=1}^{n}\left(F_j \times v_{spot}^j \times f_{j-1}\right)}{\sum_{j=1}^{n}\left(F_j \times v_{spot}^j\right)}$$

$$= \frac{\frac{500,000 \times 0.025}{1.0250} + \frac{400,000 \times 0.03502}{1.0300^2} + \frac{300,000 \times 0.06611}{1.0419^3}}{\frac{500,000}{1.0250} + \frac{400,000}{1.0300^2} + \frac{300,000}{1.0419^3}} = \frac{42,936.92}{1,130,085.76}$$

$$= \mathbf{0.037994}$$

Solution 17.17

D Section 17.05, Amortizing Swap

We can use the spot rates to determine the forward rates:

$$f_0 = 0.025$$

$$f_1 = \frac{(1+s_2)^2}{1+s_1} - 1 = \frac{1.0300^2}{1.0250} - 1 = 0.03502$$

$$f_2 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 = \frac{1.0419^2}{1.0300^2} - 1 = 0.06611$$

The swap rate is the weighted average of the forward rates, where the weights are the present values of the notional amounts:

$$c = \frac{\sum_{j=1}^{n}\left(F_j \times v_{spot}^j \times f_{j-1}\right)}{\sum_{j=1}^{n}\left(F_j \times v_{spot}^j\right)}$$

$$= \frac{\frac{500,000 \times 0.025}{1.0250} + \frac{400,000 \times 0.03502}{1.0300^2} + \frac{300,000 \times 0.06611}{1.0419^3}}{\frac{500,000}{1.0250} + \frac{400,000}{1.0300^2} + \frac{300,000}{1.0419^3}} = \frac{42,936.92}{1,130,085.76}$$

$$= 0.037994$$

The 1-year rate is 2.5%, which is less than the fixed rate of 3.7994%, so the fixed-rate payer makes a payment to the floating-rate payer. The notional amount for the second payment is 400,000:

$$\text{Net Payment to floating-rate payer} = (c - i_{float})F$$
$$= (0.037994 - 0.025) \times 400,000 = \mathbf{5,197.76}$$

Solution 17.18

A Section 17.05, Amortizing Swap

We can use the spot rates to determine the forward rates:

$$f_0 = 0.025$$

$$f_1 = \frac{(1+s_2)^2}{1+s_1} - 1 = \frac{1.0300^2}{1.0250} - 1 = 0.03502$$

$$f_2 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 = \frac{1.0419^2}{1.0300^2} - 1 = 0.06611$$

The swap rate is the weighted average of the forward rates, where the weights are the present values of the notional amounts:

$$c=\frac{\sum_{j=1}^{n}\left(F_j\times v_{spot}^{j}\times f_{j-1}\right)}{\sum_{j=1}^{n}\left(F_j\times v_{spot}^{j}\right)}$$

$$=\frac{\frac{500,000\times 0.025}{1.0250}+\frac{400,000\times 0.03502}{1.0300^2}+\frac{300,000\times 0.06611}{1.0419^3}}{\frac{500,000}{1.0250}+\frac{400,000}{1.0300^2}+\frac{300,000}{1.0419^3}}=\frac{42,936.92}{1,130,085.76}$$

$$=0.037994$$

One year later, since the spot rates have not changed, the forward rates are also unchanged. To find the current value of the swap, we compare the fixed rate with the forward rates. We make use of only the first two forward rates, because the first payment, which was based on 500,000, is now in the past:

$$\text{Value to fixed-rate payer}=\sum_{j=1}^{n}\left(F_j\times v_{spot}^{j}\times\left[f_{j-1}-c\right]\right)$$

$$=\frac{400,000(0.025-0.037994)}{1.025}+\frac{300,000(0.03502-0.037994)}{1.03^2}$$

$$=-5,070.98-839.85=\mathbf{-5,910.83}$$

Solution 17.19

B Section 17.05, Varying and Accreting Swap

We can use the forward rates to determine the present value factors corresponding to the times at which payments are made:

$$v_{spot}^{3}=P_3=\frac{1}{1.020\times 1.025\times 1.030}=0.9286$$

$$v_{spot}^{4}=P_4=\frac{1}{1.020\times 1.025\times 1.030\times 1.040}=0.8929$$

$$v_{spot}^{5}=P_5=\frac{1}{1.020\times 1.025\times 1.030\times 1.040\times 1.054}=0.8472$$

The swap rate is the weighted average of the forward rates, where the weights are the present values of the notional amounts. Since the notional amounts appear in both the numerator and the denominator below, we can assume that the notional amounts are 10, 20, and 30 without any loss of accuracy:

$$c=\frac{\sum_{j=1}^{n}\left(F_j\times v_{spot}^{j}\times f_{j-1}\right)}{\sum_{j=1}^{n}\left(F_j\times v_{spot}^{j}\right)}$$

$$=\frac{10\times 0.9286\times 0.030+20\times 0.8929\times 0.040+30\times 0.8472\times 0.054}{10\times 0.9286+20\times 0.8929+30\times 0.8472}=\frac{2.3653}{52.5591}$$

$$=\mathbf{0.0450}$$

Chapter 18: Banking

Solution 18.01

B Section 18.01, Depository Institutions

Choice **B** is false. Home mortgages, auto loans, and home equity loans are all backed by collateral: homes, autos, and homes, respectively. Therefore the loans are secured loans.

Choice E may appear to be false because a mortgage is a secured loan. But a mortgage that is guaranteed by a third party is also a guaranteed loan, so Choice E is true.

Solution 18.02

E Section 18.02, Central Banks

Choice **E** is false. The federal funds rate is the rate at which banks borrow from one another, not from the Federal Reserve. The discount rate is the rate at which banks can borrow directly from the Federal Reserve.

Solution 18.03

A Section 18.02, Central Banks

The federal funds rate is the interest rate at which banks with excess reserves lend their excess reserves to other banks. Choice **A** is the correct answer.

Solution 18.04

B Section 18.02, Central Banks

A higher federal funds rate makes it more expensive for banks to have a shortfall in their reserve accounts because the cost of borrowing to cover the shortfall is higher. Therefore, in response to a higher federal funds rate, banks write fewer loans, and the interest rates on those loans increase. Choice **B** is the correct answer.