Solutions to End of Chapter Questions

Calculator Settings

Many of the solutions below utilize the BA II Plus calculator, and those solutions are based on the following assumptions:

- The calculation method is set to the algebraic operating system (AOS). If the calculation method is currently set to the chain calculation method (Chn), then it can be changed to the AOS method with the following key strokes:

  [2nd] [FORMAT] ↓↓↓↓ [2nd] [SET] [2nd] [QUIT]

- The payments per year (P/Y) and compounding periods per year (C/Y) are set to 1. Changing the P/Y setting to 1 will automatically change the C/Y setting to 1:

  [2nd] [P/Y] 1 [ENTER] [2nd] [QUIT]

- The relevant worksheet register has been cleared prior to data entry. The keystrokes to clear the registers are:

  [2nd] [CLR TVM] to clear the time value of money worksheet

  [2nd] [CLR WORK] to clear the other worksheets

- Unless stated otherwise, the calculator is set to treat payments as occurring at the end of each period. To toggle the calculator between treating payments as occurring at the beginning of each period or the end of each period, use the following key strokes:

  [2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

If you take an actuarial exam, the exam administrator will reset your calculator prior to the exam. This means that it will then revert to using the chain calculation method, and it may change the settings for P/Y and C/Y. After it has been reset, you can easily change it back to the settings shown above. If you would like to practice, you can reset the calculator as follows:

[2nd] [RESET] [ENTER] [2nd] [QUIT]

Chapter 1: Setting the Stage

There are not any questions at the end of Chapter 1.

Chapter 2: Simple Interest and Discount

Solution 2.01

D Section 2.01, Simple Interest

Since the simple interest rate is expressed as an annual value, we use years as our unit of time. 18 months is equal to 1.5 years, so the funds accumulate from time 1.5 to time 5, which is 3.5 years. The accumulation function is based on the deposit being made at time 0, so we interpret t to be the time from the deposit until the valuation date:

$$AV_t = PV_0(1+it)$$

$$AV_{3.5} = 10,000(1+0.07\times 3.5)$$

$$AV_{3.5} = \mathbf{12,450}$$

Solution 2.02

A Section 2.02, Simple Discount

The present value of $10,000 in 8 years is:

$$PV_0 = AV_t(1-dt) = 10,000(1-0.07\times 8) = 10,000(0.44) = \mathbf{4,400}$$

Solution 2.03

E Section 2.02, Simple Interest and Discount

The accumulated values are calculated below:

$$A = AV_5 = 1,000(1+0.05\times 5) = 1,250$$

$$B = AV_5 = \frac{900}{1-0.05\times 5} = 1,200$$

$$C = AV_6 = \frac{900}{1-0.04\times 6} = 1,184.21$$

$$D = AV_4 = 1,000(1+0.06\times 4) = 1,240$$

$$E = AV_8 = \frac{1,000}{1-0.03\times 8} = 1,315.79$$

The highest accumulated value is Choice **E**.

Solution 2.04

C Section 2.03, Equivalent Simple Interest and Discount Rates

The accumulated values are equal at the end of t years:

$$100(1+0.05t) = \frac{100}{1-0.04t}$$

$$(1+0.05t)(1-0.04t) = 1$$

$$1+0.05t-0.04t+0.002t^2 = 1$$

$$0.01t-0.002t^2 = 0$$

$$0.01-0.002t = 0$$

$$t = 5$$

Eric's accumulated value at the end of 5 years is:

$$100(1+0.05t) = 100(1+0.05\times 5) = \mathbf{125}$$

Solution 2.05

A Section 2.03, Equivalent Simple Interest and Discount Rates

Let the amount of the initial loan be L. The accumulated values are equal at the end of t years:

$$L(1+it) = \frac{L}{1-dt}$$
$$1+it = \frac{1}{1-dt}$$
$$(1+it)(1-dt) = 1$$
$$1+it-dt-idt^2 = 1$$
$$it-dt-idt^2 = 0$$
$$i-d = idt$$
$$t = \frac{i-d}{id}$$

Equation I is the only valid expression for t.

Solution 2.06

B Section 2.04, Equations of Value under Simple Interest

The amount of interest earned by Ann each year is:

$$800 \times 0.06 = 48$$

Ann's interest is twice that of the interest earned by Mike:

$$48 = 2 \times X \times 0.09$$
$$X = \mathbf{266.67}$$

Solution 2.07

C Section 2.04, Equations of value under Simple Interest and Discount

The accumulated value of the fund minus the accumulated value of the loan is:

$$X = 1,000\left(1+0.06t\right) - 1,000(1-0.05t)^{-1}$$

To find the point at which X is maximized, we take the derivative of X:

$$\frac{dX}{dt} = 1,000(0.06) - 1,000(-1)(1-0.05t)^{-2}(-0.05)$$
$$= 60 - \frac{50}{(1-0.05t)^2}$$

We set the derivative equal to zero to find the local maximum:

$$0 = 60 - \frac{50}{(1-0.05t)^2}$$

$$(1-0.05t)^2 = \frac{5}{6}$$

$$1-0.05t = \pm\sqrt{\frac{5}{6}}$$

$$0.05t = 1 \pm \sqrt{\frac{5}{6}}$$

$$t = \frac{1 \pm \sqrt{\frac{5}{6}}}{0.05}$$

$$t = 38.2574 \quad \text{or } t = 1.7426$$

The question tells us that t must be between zero and 10, so we consider the value of 1.7426. To determine whether it is a local maximum or a local minimum, we take the second derivative of X and evaluate it at t = 1.7426:

$$\frac{dX}{dt} = 60 - \frac{50}{(1-0.05t)^2}$$

$$\frac{d^2X}{dt^2} = -(-2)\frac{50}{(1-0.05t)^3}(-0.05)$$

$$\frac{d^2X}{dt^2} = \frac{-5}{(1-0.05t)^3}$$

$$\frac{-5}{(1-0.05\times 1.7426)^3} = -6.5727$$

Since the second derivative is negative, there is a local maximum at 1.7426. The maximum value of X within 10 years is:

$$X = 1{,}000\left(1+0.06t\right) - 1{,}000(1-0.05t)^{-1}$$

$$= 1{,}000\left(1+0.06\times 1.7426\right) - 1{,}000(1-0.05\times 1.7426)^{-1}$$

$$= 1{,}104.55 - 1{,}095.45 = \mathbf{9.11}$$

Chapter 3: Compound Interest and Discount

Solution 3.01

D Section 3.01, Compound Interest

The number of years until the 1-year-old, 4-year-old, 6-year-old, and 9-year-old reach the age of 17 are 16, 13, 11, and 8, respectively.

The number of years until the 1-year-old, 4-year-old, 6-year-old, and 9-year-old reach the age of 22 are 21, 18, 16, and 13, respectively.

The present value of the payments matches Choice **D**:

$$A\left[v^{16}+v^{13}+v^{11}+v^{8}\right]+B\left[v^{21}+v^{18}+v^{16}+v^{13}\right]$$
$$=\left[A+Bv^{5}\right]\left[v^{16}+v^{13}+v^{11}+v^{8}\right]$$

Solution 3.02

D Section 3.01, Compound Interest

Let P be the purchase price. The two offers have the same present value:

$$0.92P(1.07)^{-9/12}=P\left(1-\tfrac{X}{100}\right)$$
$$0.8745=\left(1-\tfrac{X}{100}\right)$$
$$X=\mathbf{12.55}$$

Solution 3.03

C Section 3.01, Compound Interest

The equation of value at the outset can be used to solve for i:

$$100+200v^{n}+300v^{2n}=604.42v^{n+2}$$
$$100+200\times 0.7+300\times 0.7^{2}=604.42\times 0.7v^{2}$$
$$0.9147=v^{2}$$
$$i=\mathbf{0.0456}$$

Solution 3.04

E Section 3.01, Compound Interest

The equation of value at the outset can be used to solve for i:

$$100+200v^{n}+300v^{2n}=604.42v^{n+2}$$
$$100+200\times 0.7+300\times 0.7^{2}=604.42\times 0.7v^{2}$$
$$0.9147=v^{2}$$
$$i=0.045594$$

We now solve for n:

$$v^n = 0.7$$

$$\left(\frac{1}{1.045594}\right)^n = 0.7$$

$$n \times -\ln(1.045594) = \ln(0.70)$$

$$n = \mathbf{8.0}$$

Solution 3.05

B Section 3.01, Compound Interest

Since the amount of interest earned in Wanda's account during the 11th year is equal to the amount of interest earned in Claire's account during the 15th year, the amount in Wanda's account at the end of the 10th year must be equal to the amount in Claire's account at the end of the 14th year:

$$1,000(1+i)^{10} = 700(1+i)^{14}$$

$$\frac{10}{7} = (1+i)^4$$

$$i = 0.09327$$

The interest earned in Wanda's account in the 11th year is:

$$X = 1,000(1+i)^{10} i = 1,000(1.09327)^{10} \times 0.09327 = \mathbf{227.50}$$

Solution 3.06

E Section 3.01, Simple Interest & Compound Interest

The interest earned in Bonnie's account is the product of the original deposit, the length of time elapsed, and the simple interest rate. The interest earned in Bonnie's account during the 7th year is:

$$1,800 \times 1 \times i$$

The interest in earned in Clyde's account is the balance after 6 years times the annual effective interest rate:

$$1,000\left(1+i\right)^6 \times i$$

Setting Bonnie's interest equal to Clyde's interest allows us to solve for i:

$$1,800 \times 1 \times i = 1,000\left(1+i\right)^6 \times i$$

$$1,800 = 1,000\left(1+i\right)^6$$

$$1,800 = 1,000\left(1+i\right)^6$$

$$i = \mathbf{10.29\%}$$

Solution 3.07

A Section 3.01, Compound Interest

Equating the present value of the single payment with the present value of the set of three payments allows us to solve for T:

$$\frac{1,553.50}{1.04^{T/12}} = \frac{300}{1.04^{1/12}} + \frac{500}{1.04^{1.5}} + \frac{700}{1.04^{2}}$$

$$\frac{1,553.50}{1.04^{T/12}} = 1,417.6434$$

$$1.09583 = 1.04^{T/12}$$

$$\ln(1.09583) = \frac{T}{12} \times \ln(1.04)$$

$$T = \mathbf{27.9998}$$

Solution 3.08

C Section 3.01, Compound Interest

Let's use B_1 and B_2 to denote the balances in the account as of July 1, 2007. We are given the balance in account #1 was three times the balance in account #2:

$$B_1 = 3B_2$$

Nine years later the balance is 150,000:

$$B_1(1.03)^9 + B_2(1.05)^9 = 150,000$$

$$3B_2(1.03)^9 + B_2(1.05)^9 = 150,000$$

$$B_2 = 27,444.1395$$

The sum of the two accounts on July 1, 2007 was:

$$B_1 + B_2 = 3B_2 + B_2 = 4B_2 = 4 \times 27,444.1395 = \mathbf{109,776.56}$$

Solution 3.09

A Section 3.01, Compound Interest

There is no need to calculate the values of i or t.

Using the equality of the first and third payment streams, we can find the value of v:

$$12,000v^{12} = 8,000$$

$$v = \left(\frac{8}{12}\right)^{1/12}$$

$$v = 0.9668$$

Using the equality of the second and third payment streams, we can find v^t :

$$3,000v^t + 63,000v^{2t} = 8,000$$

$$63v^{2t} + 3v^t - 8 = 0$$

$$(21v^t + 8)(3v^t - 1) = 0$$

$$v^t = -\frac{8}{21} \quad \text{or} \quad v^t = \frac{1}{3}$$

Since the discount factor must be positive, we have:

$$v^t = \frac{1}{3}$$

The present value of 6,000 at time $(t+1)$ is:

$$6,000v^{t+1} = 6,000 \times v^t \times v = 6,000 \times \frac{1}{3} \times 0.9668 = \mathbf{1,933.55}$$

Solution 3.10

B Section 3.01, Compound Interest

Jill's accumulated amount is equal to Tom's accumulated amount at the end of 15 years:

$$40\left(1+0.05\times 15\right)+20\left(1+0.05\times(15-6)\right) = 40(1.04)^{15-n}+20(1.04)^{15-2n}$$

$$40+40\times 0.05\times 15+20+20\times 0.05\times 9 = 40(1.04)^{15-n}+20(1.04)^{15-2n}$$

$$99 = 40(1.04)^{15-n}+20(1.04)^{15-2n}$$

$$54.9712 = 40(1.04)^{-n}+20(1.04)^{-2n}$$

$$54.9712(1.04)^{2n} = 40(1.04)^{n}+20$$

$$54.9712(1.04)^{2n}-40(1.04)^{n}-20=0$$

Let's use $x = 1.04^n$ and use the quadratic formula to solve for x:

$$54.9712x^2-40x-20=0$$

$$x = \frac{40 \pm \sqrt{(-40)^2-4(54.9712)(-20)}}{2(54.9712)}$$

$$x=-0.34059 \quad \text{or} \quad x=1.06824$$

We use the positive value of x to solve for n:

$$x = 1.06824$$

$$1.04^n = 1.06824$$

$$n\ln(1.04) = \ln(1.06824)$$

$$n = \mathbf{1.6831}$$

Solution 3.11

C Section 3.03, Compound Discount

The present value of the payments is:

$$PV_0 = 10,000(1-0.065)^3+15,000(1-0.065)^5 = \mathbf{18,892.88}$$

Solution 3.12

E Section 3.03, Compound Discount

The accumulated value of the payments is:

$$AV_6 = \frac{8,000}{(1-0.07)^6}+\frac{2,000}{(1-0.07)^{6-2}} = \mathbf{15,038.56}$$

Solution 3.13

B Section 3.03, Compound Interest and Discount

The accumulated value of the deposit is:

$$AV_{4.5} = 2,000(1.08)^{4.5} = 2,827.72$$

The present value of the payment is:

$$PV_0 = 2,827.72(1-0.05)^{4.5} = \mathbf{2,244.88}$$

Solution 3.14

B Section 3.03, Compound Discount

The amount of interest earned in each fund is the accumulated value of the fund minus the original value of the fund:

$$X(1.03)^{12} - X = \frac{X}{3}\left(\frac{1}{1-d}\right)^{12} - \frac{X}{3}$$

$$(1.03)^{12} - 1 = \frac{1}{3}\left(\frac{1}{1-d}\right)^{12} - \frac{1}{3}$$

$$1.2773 = \left(\frac{1}{1-d}\right)^{12} - 1$$

$$1.07099 = \frac{1}{1-d}$$

$$d = \mathbf{0.06628}$$

Solution 3.15

B Section 3.03, Compound Discount

There is no need to calculate the value of d.

The present values of the two sets of payments are equal, allowing us to solve for the one-year discount factor:

$$169 + 169v = 196v^2 + 196v^3$$

$$169(1+v) = 196v^2(1+v)$$

$$169 = 196v^2$$

$$v = \sqrt{\frac{169}{196}}$$

$$v = \frac{13}{14}$$

The present value of the first set of payments is:

$$K = 169 + 169v = 169 + 169 \times \frac{13}{14} = \mathbf{325.93}$$

Solution 3.16

E Section 3.04, Equivalent Compound Interest and Discount

A is true:

$$i(1+i) = \frac{d}{1-d}(1+i) = \frac{d}{1-d} \times \frac{1}{v} = \frac{d}{v - vd}$$

B is true:

$$i^2 = \left(\frac{d}{1-d}\right)^2 = \left(\frac{d}{v}\right)^2 = \frac{d^2}{v^2}$$

C is true:

$$id = i(1-v) = i - iv = i - d$$

D is true:

$$i - d = i - iv = \left(\frac{1}{v} - 1\right) - iv = \frac{1}{v} - \frac{v}{v} - \frac{iv^2}{v} = \frac{1 - v - iv^2}{v}$$

E is false:

$$i + d = i + iv = i(1+v)$$
$$i(1+v) \neq i(1-v)$$

Solution 3.17

D Section 3.05, Interest Rate Conversion

The annual effective interest rate is:

$$i = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 1.01^{12} - 1 = \mathbf{0.1268}$$

Solution 3.18

E Section 3.05, Interest Rate Conversion

The two-year effective interest rate is found below:

$$\left(1 + \frac{i^{(m)}}{m}\right)^m = \left(1 + \frac{i^{(p)}}{p}\right)^p$$

$$\left(1 + \frac{i^{(1/2)}}{1/2}\right)^{1/2} = \left(1 + \frac{0.12}{12}\right)^{12}$$

$$\left(1 + \frac{i^{(1/2)}}{1/2}\right) = (1.01)^{24}$$

$$\frac{i^{(1/2)}}{1/2} = \mathbf{0.2697}$$

Solution 3.19

A Section 3.05, Simple Interest & Compound Interest

The interest earned in Patty's account is the product of the original deposit, the length of time elapsed, and the simple interest rate. The interest earned in Patty's account during the last 3 months of the 7th year is:

$$1,800 \times 0.25 \times i$$

We would normally write the nominal interest rate compounded quarterly as $i^{(4)}$, but since this question calls it *i*, we do likewise. The interest in earned in Sally's account is the balance after 6 years and 9 months, which is equal to 27 quarters, times the quarterly effective interest rate:

$$1,000\left(1 + \frac{i}{4}\right)^{27} \times \frac{i}{4}$$

Setting Patty's interest equal to Sally's interest allows us to solve for i:

$$1,800 \times 0.25 \times i = 1,000\left(1+\frac{i}{4}\right)^{27} \times \frac{i}{4}$$

$$1,800 = 1,000\left(1+\frac{i}{4}\right)^{27}$$

$$i = \mathbf{8.80\%}$$

Solution 3.20

C Section 3.05, Interest Rate Conversions

The quarterly effective interest rate can be converted to the monthly effective interest rate:

$$\frac{i^{(4)}}{4} = \frac{0.12}{4} = 0.03$$

There are 3 months in a quarter, so the monthly accumulation factor is the cube root of the quarterly accumulation factor:

$$1+\frac{i^{(12)}}{12} = 1.03^{\frac{1}{3}}$$

$$1+\frac{i^{(12)}}{12} = 1.009902$$

$$i^{(12)} = \mathbf{0.1188}$$

Solution 3.21

D Section 3.05, Nominal Interest Rates

Since the amount of interest earned in Wanda's account during the 11th year is equal to the amount of interest earned in Claire's account during the 15th year, the amount in Wanda's account at the end of the 10th year must be equal to the amount in Claire's account at the end of the 14th year:

$$1,000\left(1+\frac{i^{(12)}}{12}\right)^{10\times 12} = 700\left(1+\frac{i^{(12)}}{12}\right)^{14\times 12}$$

$$\frac{10}{7} = \left(1+\frac{i^{(12)}}{12}\right)^{4\times 12}$$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1.09327$$

The interest earned in Wanda's account in the 11th year is:

$$X = 1,000\left(1+\frac{i^{(12)}}{12}\right)^{10\times 12}\left[\left(1+\frac{i^{(12)}}{12}\right)^{12} - 1\right] = 1,000(1.09327)^{10} \times 0.09327$$

$$= \mathbf{227.50}$$

Alternative Solution: Since the two accounts earn the same effective interest rate compounded monthly, they must also earn the same equivalent annual effective interest rate. Therefore, as shown below, we can answer the question without reference to the monthly effective interest rate.

Since the amount of interest earned in Wanda's account during the 11th year is equal to the amount of interest earned in Claire's account during the 15th year, the amount in Wanda's account at the end of the 10th year must be equal to the amount in Claire's account at the end of the 14th year:

$$1,000(1+i)^{10} = 700(1+i)^{14}$$
$$\frac{10}{7} = (1+i)^4$$
$$i = 0.09327$$

The interest earned in Wanda's account in the 11th year is:

$$X = 1,000(1+i)^{10} i = 1,000(1.09327)^{10} \times 0.09327 = \mathbf{227.50}$$

Solution 3.22

D Section 3.05, Nominal Interest Rates

Although we usually use i to denote an annual effective interest rate, this question uses i to denote the annual interest rate that is compounded semiannually. We would usually use $i^{(2)}$ to denote this interest rate.

Sam's interest is based on the balance at the end of 13 6-month periods, and Dennis' interest is based on the initial deposit:

$$D \times \left(1+\frac{i}{2}\right)^{13} \times \frac{i}{2} = 2D \times \frac{i}{2}$$
$$\left(1+\frac{i}{2}\right)^{13} = 2$$
$$i = \mathbf{10.95\%}$$

Solution 3.23

E Section 3.05, Nominal Interest Rates

At the outset, the present value of Heidi's loan is equal to the present value of Adam's loan:

$$\frac{1,400}{\left(1+\frac{i^{(2)}}{2\times 2}\right)^{40}} = \frac{2,000}{\left(1+\frac{i^{(2)}}{2}\right)^{40}}$$
$$\frac{\left(1+\frac{i^{(2)}}{2}\right)}{\left(1+\frac{i^{(2)}}{4}\right)} = \left(\frac{2,000}{1,400}\right)^{1/40}$$
$$\frac{\left(1+\frac{i^{(2)}}{2}\right)}{\left(1+\frac{i^{(2)}}{4}\right)} = 1.008957$$
$$1+0.5i^{(2)} = 1.008957 + 0.252239i^{(2)}$$
$$0.247761i^{(2)} = 0.008957$$
$$i^{(2)} = 0.03615$$

We can now find the present value of Heidi's loan:

$$L = \frac{1,400}{\left(1+\frac{i^{(2)}}{4}\right)^{40}} = \frac{1,400}{\left(1+\frac{0.03615}{4}\right)^{40}} = \mathbf{976.86}$$

Solution 3.24

D Section 3.05, Nominal Interest Rates

Interest is credited only at the end of each interest conversion period, so Michelle receives interest only every 3 months. Therefore, the moment at which Michelle's account is at least double the amount in Lucy's account will occur on some multiple of 3 months.

Let t be the number of 3-month periods until Michelle's account is at least double the amount in Lucy's account. We need to find the minimum integer value of t that satisfies the following:

$$\left(1+\frac{0.12}{4}\right)^{t} \geq 2\left(1+\frac{0.06}{12}\right)^{3t}$$

$$t\ln(1.03) \geq \ln(2) + 3t\ln(1.005)$$

$$t \geq 47.4883$$

The smallest integer that satisfies the equation above is t = 48. The number of months in 48 three-month intervals is:

$$48 \times 3 = \mathbf{144}$$

Solution 3.25

B Section 3.05, Nominal Interest Rates

The monthly effective interest rate is:

$$\frac{0.09}{12} = 0.0075$$

The equation of value that equates the value of the deposits with $5,900 six years from today is:

$$1,500 \times 1.0075^{72-n} + 3,000 \times 1.0075^{72-2n} = 5,900$$

$$1,500 \times 1.0075^{-n} + 3,000 \times 1.0075^{-2n} = 5,900 \times 1.0075^{-72}$$

$$3,000X^2 + 1,500X - 3,445.1494 = 0 \quad \text{where: } X = 1.0075^{-n}$$

We can use the quadratic formula to solve for X:

$$X = \frac{-1,500 \pm \sqrt{1,500^2 - 4(3,000)(-3,445.1494)}}{2 \times 3,000}$$

$$X = -1.3504 \text{ or } X = 0.8504$$

Using the positive value of X, we have the maximum possible value of n:

$$X = 1.0075^{-n}$$

$$0.8504 = 1.0075^{-n}$$

$$n = 21.6872$$

Since n must be less than or equal to 21.6872, the maximum integral value of n is **21**.

Solution 3.26

D Section 3.06, Nominal Interest and Discount Rates

The accumulated value of the deposit is:

$$AV_{4.5} = 2,000\left(1+\frac{0.08}{2}\right)^{4.5\times 2} = 2,000(1.04)^{9} = 2,846.62$$

The present value of the payment is:

$$PV_0 = 2,846.62\left(1-\frac{0.05}{12}\right)^{4.5\times 12} = 2,846.62(0.9958)^{54} = \mathbf{2,272.01}$$

Solution 3.27

D Section 3.06, Nominal Discount Rates

The monthly effective discount rate is:

$$\frac{0.09}{12} = 0.0075$$

The equation of value that equates the value of the deposits with $5,900 six years from today is:

$$1,500\times\left(\frac{1}{1-0.0075}\right)^{72-n} + 3,100\times\left(\frac{1}{1-0.0075}\right)^{72-2n} = 5,900$$

$$1,500\times 0.9925^{n-72} + 3,100\times 0.9925^{2n-72} = 5,900$$

$$1,500\times 0.9925^{n} + 3,100\times 0.9925^{2n} = 5,900\times 0.9925^{72}$$

$$3,100X^2 + 1,500X - 3,431.2244 = 0 \quad \text{where: } X = 0.9925^{n}$$

We can use the quadratic formula to solve for X:

$$X = \frac{-1,500 \pm \sqrt{1,500^2 - 4(3,100)(-3,431.2244)}}{2\times 3,100}$$

$$X = -1.3215 \text{ or } X = 0.8376$$

Using the positive value of X, we have the maximum possible value of n:

$$X = 0.9925^{n}$$

$$0.8376 = 0.9925^{n}$$

$$n = 23.5412$$

Since n must be less than or equal to 23.5412, the maximum integral value of n is **23**.

Solution 3.28

C Section 3.07, Interest Rate Conversions

The monthly accumulation factor is equal to the inversion of the monthly discount factor:

$$1+\frac{i^{(12)}}{12} = \left(1-\frac{d^{(12)}}{12}\right)^{-1}$$

$$1+\frac{i^{(12)}}{12} = \left(1-\frac{0.22}{12}\right)^{-1}$$

$$i^{(12)} = \mathbf{0.2241}$$

Solution 3.29

C Section 3.07, Interest Rate Conversions

The interest rate and the discount rate produce the same one-year accrual factor:

$$\left(1+\frac{i^{(m)}}{m}\right)^{m}=\left(1-\frac{d^{(p)}}{p}\right)^{-p}$$

$$\left(1+\frac{0.14}{12}\right)^{12}=\left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$\left(1+\frac{0.14}{12}\right)^{-3}=1-\frac{d^{(4)}}{4}$$

$$d^{(4)}=\mathbf{0.1368}$$

Solution 3.30

B Section 3.07, Nominal Discount Rates

We can use the ratio of A to B to find d:

$$\frac{A}{B}=\left(\frac{51}{52}\right)^{16}$$

$$\frac{50\left(1-\frac{d}{4}\right)^{-4\times4}}{50\left(1-\frac{d}{2}\right)^{-8\times2}}=\left(\frac{51}{52}\right)^{16}$$

$$\frac{\left(1-\frac{d}{2}\right)}{\left(1-\frac{d}{4}\right)}=\frac{51}{52}$$

$$52-26d=51-12.75d$$

$$1=13.25d$$

$$d=0.07457$$

The value of d convertible quarterly is equivalent to an annual effective interest rate of i:

$$\left(1-\frac{d}{4}\right)^{-4}=1+i$$

$$\left(1-\frac{0.07547}{4}\right)^{-4}=1+i$$

$$i=\mathbf{0.0792}$$

Chapter 4: Constant Force of Interest

Solution 4.01

B Section 4.02, Force of Interest

The force of interest is:

$$r = \ln(1.080) = \mathbf{0.07696}$$

Solution 4.02

A Section 4.02, Force of Interest

The force of interest is:

$$r = \ln\left[\left(1 + \frac{0.10}{12}\right)^{12}\right] = \mathbf{0.09959}$$

Solution 4.03

C Section 4.02, Force of Interest

The force of interest is:

$$r = \ln\left[\left(1 - \frac{0.09}{4}\right)^{-4}\right] = \mathbf{0.09103}$$

Solution 4.04

B Section 4.02, Force of Interest

The force of interest is:

$$r = \ln\left[(1 - 0.01)^{-12}\right] = \mathbf{0.1206}$$

Solution 4.05

A Section 4.02, Force of Interest

The force of interest is:

$$r = \ln\left[\left(1 + \frac{0.11}{\frac{1}{2}}\right)^{1/2}\right] = \mathbf{0.09943}$$

Solution 4.06

Section 4.02, Force of Interest

a. The annual effective interest rate is:

$$i = e^{0.06} - 1 = \mathbf{0.06184}$$

b. The monthly effective interest rate is:

$$\frac{i^{(12)}}{12} = e^{0.06/12} - 1 = \mathbf{0.00501}$$

c. The annual interest rate compounded monthly is:

$$i^{(12)} = 12\left(e^{0.06/12} - 1\right) = 12 \times 0.00501 = \mathbf{0.06015}$$

d. The annual effective discount rate is:

$$d = 1 - e^{-0.06} = \mathbf{0.05824}$$

e. The quarterly effective discount rate is:

$$\frac{d^{(4)}}{4} = 1 - e^{-0.06/4} = \mathbf{0.01489}$$

e. The annual discount rate convertible quarterly is:

$$d^{(4)} = 4\left(1 - e^{-0.06/4}\right) = 4 \times 0.01489 = \mathbf{0.05955}$$

Solution 4.07

B Section 4.02, Force of Interest

The present value is:

$$3,000e^{-0.07\times2} + 8,000e^{-0.07\times5} + 10,000e^{-0.07\times7}$$
$$= 3,000 \times 0.8694 + 8,000 \times 0.7047 + 10,000 \times 0.6126$$
$$= \mathbf{14,371.84}$$

Solution 4.08

D Section 4.02, Force of Interest

The balance at the end of 10 years is:

$$2,000e^{0.07\times10} - 1,500e^{0.07\times6} + 8,000e^{0.07\times4}$$
$$= 2,000 \times 2.0138 - 1,500 \times 1.5220 + 8,000 \times 1.3231$$
$$= \mathbf{12,329.60}$$

Solution 4.09

E Section 4.02, Force of Interest

The original force of interest is:

$$r = \ln\left[\left(1 + \frac{0.07}{4}\right)^4\right] = 0.06939$$

One half of the original force of interest is:

$$0.5 \times 0.06939 = 0.03470$$

The new annual interest rate compounded quarterly is:

$$i^{(4)} = 4 \times (e^{0.03470\times0.25} - 1) = \mathbf{0.0348}$$

Solution 4.10

A Section 4.02, Force of Interest

The equation of value at the outset can be used to solve for *r*:

$$100 + 200v^{n} + 300v^{2n} = 604.42v^{n+2}$$

$$100 + 200 \times 0.7 + 300 \times 0.7^{2} = 604.42 \times 0.7v^{2}$$

$$0.9147 = v^{2}$$

$$1 + i = 1.04559$$

$$\ln(1+i) = \mathbf{0.0446}$$

Solution 4.11

C Section 4.02, Force of Interest

The equation of value at the outset can be used to solve for v^n:

$$100 + 200v^{n} + 300v^{2n} = 600v^{n}$$

$$300v^{2n} - 400v^{n} + 100 = 0$$

$$3v^{2n} - 4v^{n} + 1 = 0$$

$$(3v^{n} - 1)(v^{n} - 1) = 0$$

$$v^{n} = 1 \quad \text{or} \quad v^{n} = \frac{1}{3}$$

The annual force of interest is greater than zero, so v^n must be 1/3:

$$v^{n} = \frac{1}{3}$$

$$e^{-0.1221n} = \frac{1}{3}$$

$$-0.1221n = \ln\left(\frac{1}{3}\right)$$

$$n = \mathbf{9.00}$$

Solution 4.12

E Section 4.02, Force of Interest

For a given force of interest, the equivalent nominal interest rate falls as its compounding frequency increases. The expression in Choice E can be rewritten as:

$$i^{(1/2)} < i^{(1)}$$

Increasing the compounding frequency from every other year to every year will result in a higher force of interest unless the interest rate with the higher compounding frequency is lower than the interest rate with the lower compounding frequency. Therefore, Choice **E** is false.

Solution 4.13

E Section 4.02, Force of Interest

The equation of value at the end of 12 years and 1 month can be used to solve for the force of interest:

$$200\left(1+\frac{0.075}{4}\right)^{4\times\left(12+\frac{1}{12}\right)} = 220e^{\delta\times\left(12+\frac{1}{12}\right)}$$

$$490.8684 = 220e^{12.0833\delta}$$

$$\ln(490.8684) = \ln(220) + 12.0833\delta$$

$$0.8025 = 12.0833\delta$$

$$\delta = \mathbf{0.06642}$$

Solution 4.14

A Section 4.02, Force of Interest

At the end of 5 years the amount in Suzie's account is:

$$200(1+5\times 0.05) = 250$$

The amount in both accounts is the same at the end of 5 years:

$$250 = 220e^{5\delta}$$

$$\ln(250) = \ln(220) + 5\delta$$

$$\delta = \mathbf{0.02557}$$

Solution 4.15

D Section 4.02, Force of Interest

The derivative of the force of interest with respect to the annual effective interest rate is:

$$\frac{d}{di}\delta = \frac{d}{di}\ln(1+i) = \frac{1}{1+i} = v$$

The derivative of the annual effective interest rate with respect to the annual effective discount rate is:

$$\frac{d}{dd}i = \frac{d}{dd}\left(\frac{d}{1-d}\right) = \frac{(1-d)\times 1 - d\times(-1)}{(1-d)^2} = \frac{(1-d)+d}{(1-d)^2} = \frac{1}{(1-d)^2} = \frac{1}{v^2}$$

The product is:

$$\left[\frac{d}{di}\delta\right]\times\left[\frac{d}{dd}i\right] = v\times\frac{1}{v^2} = \frac{1}{v} = \mathbf{1+i}$$

Chapter 5: Varying Rates

Solution 5.01

A Section 5.01, Varying Compound Interest

The present value is:

$$PV_0 = 100(1.07)^{-1.5}\left(1+\frac{0.08}{4}\right)^{-1\times 4}\left(1+\frac{0.06}{12}\right)^{-1.5\times 12} = \mathbf{76.3018}$$

Solution 5.02

C Section 5.01, Varying Compound Interest

The equation of value can be used to find the level equivalent interest rate:

$$Deposit \times (1.07)^{1.5}(1.02)^4(1.005)^{18} = Deposit \times \left(1+\frac{i^{(2)}}{2}\right)^8$$

$$(1.07)^{1.5}(1.02)^4(1.005)^{18} = \left(1+\frac{i^{(2)}}{2}\right)^8$$

$$i^{(2)} = \mathbf{0.0688}$$

Solution 5.03

B Section 5.02, Varying Discount Rates

The present value is:

$$PV_0 = 100(1-0.07)^{1.5}\left(1-\frac{0.08}{4}\right)^{1\times 4}\left(1-\frac{0.06}{12}\right)^{1.5\times 12} = \mathbf{75.5865}$$

Solution 5.04

B Section 5.02, Varying Discount Rates

The accumulated value is:

$$100(0.93)^{-1.5}(0.98)^{-4}(0.995)^{-18} + 50(0.98)^{-2}(0.995)^{-18}$$
$$= 132.2988 + 56.9774 = \mathbf{189.2761}$$

Solution 5.05

B Section 5.02, Varying Discount Rates

The equation of value at time 5 can be used to solve for d:

$$2\times 100(1.10)^2(1.04)^6 = 150\left(\frac{1}{0.99}\right)^{36}\left(\frac{1}{1-d}\right)^2$$

$$306.2072 = 215.3894\left(\frac{1}{1-d}\right)^2$$

$$d = \mathbf{0.1613}$$

Solution 5.06

A Section 5.03, Varying Force of Interest

The present value is:

$$PV_0 = 100 \times e^{-1.5\times 0.07} e^{-0.08} e^{-1.5\times 0.06} = 100 \times e^{-0.275} = \mathbf{75.9572}$$

Solution 5.07

D Section 5.03, Varying Force of Interest

The accumulated value is:

$$100 \times e^{1.5\times 0.07} e^{0.08} e^{1.5\times 0.06} + 50 \times e^{0.5\times 0.08} e^{1.5\times 0.06}$$
$$= 100 \times e^{0.275} + 50 \times e^{0.13} = \mathbf{188.5945}$$

Solution 5.08

Section 5.03, Varying Force of Interest

The accumulated value function and its derivative are:

$$AV_t = D \times (1 + 0.05t)$$
$$\frac{d\left(AV_t\right)}{dt} = 0.05D$$

The force of interest at time t is:

$$r_t = \frac{\frac{d\left(AV_t\right)}{dt}}{AV_t} = \frac{0.05D}{D \times (1 + 0.05t)} = \frac{0.05}{1 + 0.05t}$$

a. $r_{\frac{1}{12}} = \dfrac{0.05}{1 + 0.05 \times \frac{1}{12}} = \mathbf{0.04979}$

b. $r_1 = \dfrac{0.05}{1 + 0.05 \times 1} = \mathbf{0.04762}$

c. $r_5 = \dfrac{0.05}{1 + 0.05 \times 5} = \mathbf{0.0400}$

d. $r_{10} = \dfrac{0.05}{1 + 0.05 \times 10} = \mathbf{0.0333}$

Solution 5.09

Section 5.03, Varying Force of Interest

The accumulated value function and its derivative are:

$$AV_t = D \times (1 - 0.05t)^{-1}$$
$$\frac{d\left(AV_t\right)}{dt} = -D \times (1 - 0.05t)^{-2}(-0.05) = 0.05D \times (1 - 0.05t)^{-2}$$

The force of interest at time t is:

$$r_t = \frac{\frac{d\left(AV_t\right)}{dt}}{AV_t} = \frac{0.05D \times (1 - 0.05t)^{-2}}{D \times (1 - 0.05t)^{-1}} = \frac{0.05}{1 - 0.05t}$$

a. $r_{\frac{1}{12}} = \dfrac{0.05}{1 - 0.05 \times \frac{1}{12}} = \mathbf{0.05021}$

b. $r_1 = \dfrac{0.05}{1-0.05\times 1} = \mathbf{0.05263}$

c. $r_5 = \dfrac{0.05}{1-0.05\times 5} = \mathbf{0.0667}$

d. $r_{10} = \dfrac{0.05}{1-0.05\times 10} = \mathbf{0.1000}$

Solution 5.10

C Section 5.03, Varying Force of Interest

The present value at time 5 of the 12,000 payment is:

$$PV_5 = AV_8 e^{-\int_5^8 r_s ds} = 12,000\times e^{-\int_5^8 \frac{1}{2+s}ds} = 12,000\times e^{-\ln(2+s)\big|_5^8}$$

$$= 12,000\times e^{\ln(7)-\ln(10)} = 12,000\times\frac{7}{10} = 8,400$$

The equation of value at time 5 can be used to solve for X:

$$5,000(1.05)^5 + X(1.05)^3 = 8,400$$

$$X = \mathbf{1,743.74}$$

Solution 5.11

B Section 5.03, Varying Force of Interest

The force of interest at time t for Fund X is:

$$r_t^X = \frac{\dfrac{d\left(AV_t\right)}{dt}}{AV_t} = \frac{0.5}{1+0.5t}$$

The force of interest at time t for Fund Y is:

$$r_t^Y = \frac{\dfrac{d\left(AV_t\right)}{dt}}{AV_t} = \frac{t}{1+0.5t^2}$$

Setting the two equal allows us to determine the time at which the two forces of interest are equal:

$$r_t^X = r_t^Y$$

$$\frac{0.5}{1+0.5t} = \frac{t}{1+0.5t^2}$$

$$0.5 + 0.25t^2 = t + 0.5t^2$$

$$0 = 0.25t^2 + t - 0.50$$

We use the quadratic formula to solve for t:

$$t = \frac{-1\pm\sqrt{1^2-4(0.25)(-0.5)}}{2\times 0.25}$$

$$t = -4.4495 \quad\text{or}\quad t = 0.4495$$

Discarding the negative solution, we have:

$$t = \mathbf{0.4495}$$

Solution 5.12

A Section 5.03, Varying Force of Interest

The accumulated value in Fund X at time *t* is:

$$X(t) = AV_t = e^{\int_0^t r_s ds} = e^{\int_0^t \frac{1}{1+s} ds} = e^{\ln(1+s)|_0^t} = e^{\ln(1+t)-\ln(1)} = e^{\ln\left(\frac{1+t}{1}\right)} = 1 + t$$

The accumulated value in Fund Y at time *t* is:

$$Y(t) = AV_t = e^{\int_0^t r_s ds} = e^{\int_0^t \frac{10s}{1+5s^2} ds}$$

Let's use the following substitution:

$$u = 1 + 5s^2$$
$$du = 10sds$$

The accumulated value in Fund Y at time *t* can now be written as:

$$Y(t) = e^{\int_0^t \frac{10s}{1+5s^2} ds} = e^{\int_{s=0}^{s=t} \frac{1}{u} du} = e^{\ln(u)|_{s=0}^{s=t}} = e^{\ln(1+5s^2)|_{s=0}^{s=t}} = e^{\ln(1+5t^2)-\ln(1)} = e^{\ln\left(\frac{1+5t^2}{1}\right)}$$
$$= 1 + 5t^2$$

We can find the maximum of $H(t)$ by setting its derivative equal to zero:

$$H(t) = X(t) - Y(t)$$
$$H(t) = 1 + t - (1 + 5t^2)$$
$$H'(t) = 1 - 10t$$
$$1 - 10t = 0$$
$$t = \mathbf{0.10}$$

For the sake of thoroughness, we note that the second derivative of $H(t)$ is negative at $t = 0.1$, indicating that $H(0.1)$ is a maximum:

$$H''(t) = -10$$

Solution 5.13

D Section 5.03, Varying Force of Interest

The equation of value is:

$$1,000e^{\int_0^5 r_s ds} = 1,000(1 + i)^5$$
$$e^{\int_0^5 \frac{1}{3(1+s)^3} ds} = (1 + i)^5$$

To evaluate the integral, let's use the following substitution:

$$u = (1 + s)$$
$$du = ds$$

The integral is:

$$\int_0^5 \frac{1}{3(1+s)^3} ds = \int_{s=0}^{s=5} \frac{1}{3} u^{-3} du = \left[\frac{1}{3} \times \frac{u^{-2}}{-2}\right]_{s=0}^{s=5} = \left[-\frac{1}{6} \times \frac{1}{(1+s)^2}\right]_{s=0}^{s=5}$$
$$= -\frac{1}{6} \times \left[\frac{1}{36} - 1\right] = 0.16204$$

We can now use the equation of value to solve for i:

$$e^{\int_0^5 \frac{1}{3(1+s)^3}ds} = (1+i)^5$$

$$e^{0.16204} = (1+i)^5$$

$$1.17590 = (1+i)^5$$

$$i = \mathbf{0.032938}$$

Solution 5.14

C Section 5.03, Varying Force of Interest

The accumulated value in Fund B at time t is:

$$AV_t = e^{\int_0^t r_s ds}$$

Let's evaluate the integral in the expression above:

$$\int_0^t \frac{2s^3+6s}{s^4+6s^2+9}ds = \int_0^t \frac{2s(s^2+3)}{(s^2+3)^2}ds = \int_0^t \frac{2s}{(s^2+3)}ds$$

Let's use the following substitution:

$$u = s^2 + 3$$

$$du = 2sds$$

The integral is:

$$\int_0^t \frac{2s}{(s^2+3)}ds = \int_{s=0}^{s=t} \frac{1}{u}du = \ln(u)\Big|_{s=0}^{s=t} = \ln(s^2+3)\Big|_0^t = \ln(t^2+3) - \ln(3) = \ln\left(\frac{t^2+3}{3}\right)$$

The accumulated value of Fund B at time t is:

$$AV_t = e^{\int_0^t r_s} = \frac{t^2+3}{3}$$

At time 1, the accumulated value in Fund B is equal to the accumulated value in Fund A:

$$\frac{t^2+3}{3} = 1 + it \quad \text{when } t = 1$$

$$\frac{1^2+3}{3} = 1 + i \times 1$$

$$\frac{4}{3} = 1 + i$$

$$i = \frac{1}{3}$$

Let $H(t)$ be the difference between Fund A and Fund B. We can find the maximum of $H(t)$ by setting its derivative equal to zero:

$$H(t) = 1 + it - \frac{t^2 + 3}{3}$$

$$H(t) = 1 + \frac{t}{3} - \frac{t^2 + 3}{3}$$

$$H'(t) = \frac{1}{3} - \frac{2t}{3}$$

$$\frac{1}{3} - \frac{2t}{3} = 0$$

$$1 - 2t = 0$$

$$2t = 1$$

$$t = \mathbf{0.5}$$

For the sake of thoroughness, we note that the second derivative of $H(t)$ is negative at $t = 0.5$, indicating that $H(0.5)$ is a maximum:

$$H''(t) = -\frac{2}{3}$$

Solution 5.15

E Section 5.03, Varying Force of Interest

The integral of the force of interest from time 4 to time 8 is:

$$\int_4^8 \delta_t dt = \int_4^8 0.001(t^2 - t)dt = 0.001\left(\frac{t^3}{3} - \frac{t^2}{2}\right)\Bigg|_4^8 = 0.001\left[\left(\frac{8^3}{3} - \frac{8^2}{2}\right) - \left(\frac{4^3}{3} - \frac{4^2}{2}\right)\right]$$

$$= 0.001\left[138.6667 - 13.3333\right] = 0.1253$$

The accumulated value at time 8 is:

$$100 \times e^{0.05\times4} \times e^{\int_4^8 \delta_t dt} = 100 \times e^{0.20} \times e^{0.1253} = \mathbf{138.45}$$

Solution 5.16

C Section 5.03, Varying Force of Interest

The integral of the force of interest from time 6 to time 10 is:

$$\int_6^{10} \delta_t dt = \int_6^{10} 0.002(t^2 + t)dt = 0.002\left(\frac{t^3}{3} + \frac{t^2}{2}\right)\Bigg|_6^{10}$$

$$= 0.002\left[\left(\frac{10^3}{3} + \frac{10^2}{2}\right) - \left(\frac{6^3}{3} + \frac{6^2}{2}\right)\right] = 0.002\left[383.3333 - 90\right]$$

$$= 0.5867$$

The present value at time 2 is:

$$100 \times e^{-\int_6^{10} \delta_t dt} \times e^{-0.02\times(6-2)} = 100 \times e^{-0.5867} \times e^{-0.08} = \mathbf{51.3417}$$

Solution 5.17

D Section 5.03, Varying Force of Interest

The interest accumulation factors must be the same for Marcia and Jan over the course of 5 years. The interest accumulation factor for Jan is:

$$\exp\left[\int_0^5 \frac{1}{K+0.20t}\,dt\right] = \exp\left[\left.\frac{\ln(K+0.20t)}{0.20}\right|_0^5\right] = \exp\left[5\times\ln(K+1) - 5\times\ln(K)\right]$$

$$= \exp\left[5\times\ln\left(\frac{K+1}{K}\right)\right] = \left(\frac{K+1}{K}\right)^5$$

The interest accumulation factor for Jan is equal to the interest accumulation factor for Marcia:

$$\left(\frac{K+1}{K}\right)^5 = \left(1+\frac{K}{36}\right)^5$$

$$\frac{K+1}{K} = 1+\frac{K}{36}$$

$$K+1 = K+\frac{K^2}{36}$$

$$1 = \frac{K^2}{36}$$

$$K = 6$$

Jan's accumulated value at the end of 5 years is:

$$100\left(\frac{K+1}{K}\right)^5 = 100\left(\frac{7}{6}\right)^5 = \mathbf{216.14}$$

Solution 5.18

B Section 5.03, Varying Force of Interest

At time 4, before the deposit of X, the value of the fund is:

$$100e^{\int_0^4 \frac{t^3}{2{,}000}dt} = 100e^{\left.\frac{t^4}{8{,}000}\right|_0^4} = 100e^{0.032}$$

The accumulation factor from time 4 to time 8 is:

$$e^{\int_4^8 \frac{t^3}{2{,}000}dt} = e^{\left.\frac{t^4}{8{,}000}\right|_4^8} = e^{0.512-0.032} = e^{0.48}$$

The interest earned from time 4 to time 8 is equal to X:

$$\left(100e^{0.032}+X\right)\left[e^{0.48}-1\right] = X$$

$$\left(100e^{0.032}+X\right)\left[0.6161\right] = X$$

$$63.6108 = 0.3839X$$

$$X = \mathbf{165.6851}$$

Solution 5.19

D Section 5.03, Varying Force of Interest

The equation of value is:

$$De^{\int_0^t \delta_s ds} = 3D$$

$$\exp\left[\int_0^t \frac{s^2}{4+s^3} ds\right] = 3$$

$$\int_0^t \frac{s^2}{4+s^3} ds = \ln 3$$

To evaluate the integral, let's use the following substitution:

$$u = 4 + s^3$$

$$du = 3s^2 ds$$

The integral is:

$$\int_0^t \frac{s^2}{4+s^3} ds = \frac{1}{3}\int_0^t \frac{1}{4+s^3} 3s^2 ds = \frac{1}{3}\int_{s=0}^{s=t} \frac{1}{u} du = \frac{1}{3} \times \ln u\Big|_{s=0}^{s=t} = \frac{1}{3} \times \ln\left(4 + s^3\right)\Big|_{s=0}^{s=t}$$

$$= \frac{1}{3} \times \left[\ln\left(4 + t^3\right) - \ln(4)\right] = \frac{1}{3} \times \ln\left(\frac{4 + t^3}{4}\right)$$

We can now solve for t:

$$\int_0^t \frac{s^2}{4+s^3} ds = \ln 3$$

$$\frac{1}{3} \times \ln\left(\frac{4 + t^3}{4}\right) = \ln 3$$

$$\left(\frac{4 + t^3}{4}\right)^{1/3} = 3$$

$$t = \mathbf{4.7027}$$

Solution 5.20

E Section 5.03, Varying Force of Interest

The present value is:

$$800e^{-\int_0^8 \delta_s ds}$$

To evaluate the integral, let's use the following substitution:

$$u = 4 + \frac{s^3}{180}$$

$$du = \frac{s^2}{60} ds$$

The integral is:

$$\int_0^t \frac{\frac{s^2}{120}}{4 + \frac{s^3}{180}} ds = 0.5\int_0^t \frac{1}{4 + \frac{s^3}{180}} \frac{s^2}{60} ds = 0.5\int_{s=0}^{s=5} \frac{1}{u} du = 0.5 \times \ln u\Big|_{s=0}^{s=5}$$

$$= 0.5 \times \ln\left(4 + \frac{s^3}{180}\right)\Big|_{s=0}^{s=5} = 0.5 \times \left[\ln\left(4 + \frac{125}{180}\right) - \ln(4)\right] = \ln\left(1.0833\right)$$

The present value is:

$$800e^{-\int_0^8 \delta_s ds} = 800e^{-\ln(1.0833)} = \frac{800}{1.0833} = \mathbf{738.46}$$

Solution 5.21

E Section 5.04, Mix of Varying Rates

The accumulated value is:

$$100\left(1-\frac{0.06}{12}\right)^{-12}\left(1+\frac{0.07}{4}\right)^{4} e^{0.10} = 100(0.995)^{-12}(1.0175)^{4}e^{0.10} = \mathbf{125.80}$$

Solution 5.22

A Section 5.04, Mix of Varying Rates

The present value is:

$$100\left(1-\frac{0.06}{12}\right)^{12}\left(1+\frac{0.07}{4}\right)^{-4} e^{-0.10} = 100(0.995)^{12}(1.0175)^{-4}e^{-0.10} = \mathbf{79.49}$$

Solution 5.23

C Section 5.04, Mix of Varying Rates

The equation of value at the end of 6 years can be used to find d:

$$500(1-d)^{-4}\left(1+\frac{d}{2}\right)^{2\times 2} = 767$$

$$\left(1+\frac{d}{2}\right)^{4} = 1.534(1-d)^{4}$$

$$1+0.5d = 1.1129(1-d)$$

$$1.6129d = 0.1129$$

$$d = \mathbf{0.07}$$

Solution 5.24

C Section 5.04, Mix of Varying Rates

The equation of value at the end of 20 years can be used to find d:

$$100\left(1-\frac{d}{2}\right)^{-2\times 7}\left(1+\frac{0.07}{12}\right)^{13\times 12} + 50\left(1+\frac{0.07}{12}\right)^{10\times 12} = 500$$

$$100\left(1-\frac{d}{2}\right)^{-14}\left(1+\frac{0.07}{12}\right)^{13\times 12} = 399.5169$$

$$\left(1-\frac{d}{2}\right)^{-14} = 1.6124$$

$$d = \mathbf{0.0671}$$

Solution 5.25

A Section 5.04, Mix of Varying Rates

Although we usually use d to denote an annual effective discount rate, this question uses d to denote the annual discount rate that is compounded quarterly. We would usually use $d^{(4)}$ *to denote this interest rate.*

The equation of value can be used to find the value of d:

$$\frac{20}{\left(1-\frac{d}{4}\right)^{5\times 4}}(1.02)^{15\times 2}+45(1.02)^{10\times 2}=110$$

$$\left(1-\frac{d}{4}\right)^{20}=0.83991$$

$$d=\mathbf{3.47\%}$$

