Solutions to Interest Theory Sample Questions

Solution 1

C Chapter 4, Interest Rate Conversion

After 7.25 years, the value of each account is the same:

$$100\left(1+\frac{0.04}{2}\right)^{7.25\times 2} = 100e^{\delta\times 7.25}$$

$$1.3326 = e^{\delta\times 7.25}$$

$$\ln(1.3326) = 7.25\delta$$

$$\delta = \mathbf{0.0396}$$

Solution 2

E Chapter 7, Level Annuities

Let j be the effective interest rate for an interval of 4 years:

$$j = (1+i)^4 - 1$$

There are 10 4-year intervals in 40 years, and there are 5 4-year intervals in 20 years. Therefore, the accumulated value at the end of 10 intervals is equal to 5 times the accumulated value at the end of 5 intervals:

$$100\ddot{s}_{\overline{10}|j} = 5\times 100\ddot{s}_{\overline{5}|j}$$

$$\frac{(1+j)^{10}-1}{j/(j+1)} = 5\times\frac{(1+j)^5-1}{j/(j+1)}$$

$$\frac{(1+j)^{10}-1}{(1+j)^5-1} = 5$$

$$(1+j)^5 + 1 = 5$$

$$(1+j)^5 = 4$$

$$j = 0.3195$$

The accumulated amount at the end of 40 years is:

$$X = 100\ddot{s}_{\overline{10}|0.3195} = 100\times\frac{1.3195^{10}-1}{0.3195}\times 1.3195 = \mathbf{6{,}194.72}$$

The BA-II Plus can be used as follows:

4 [y^x] 0.2 [=] [–] 1 [=] [×] 100 [=] [*I/Y*]

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

10 [*N*] 100 [*PMT*] [CPT] [*FV*]

Result is –6,194.72. Solution is **6,194.72**.

Solution 3

C Chapter 3, Simple Interest

Eric and Mike earn the same amount of interest during the last 6 months of the 8th year:

$$100\left(1+\frac{i}{2}\right)^{15}\times\frac{i}{2}=200\times\frac{i}{2}$$

$$\left(1+\frac{i}{2}\right)^{15}=2$$

$$i=\mathbf{0.946}$$

Solution 4

A Chapter 12, Sinking Fund

The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan:

$$1{,}627.45-10{,}000\times 0.10=627.45$$

The accumulated value of the deposits is:

$$627.45\times s_{\overline{10}|0.14}=627.45\times\frac{1.14^{10}-1}{0.14}=627.45\times 19.3373=12{,}133.1858$$

After repaying the loan, the balance is:

$$12{,}133.1858-10{,}000=\mathbf{2{,}133.19}$$

The BA-II Plus can be used to answer this question:

10 [*N*] 14 [*I*/*Y*] 627.45 [*PMT*] [CPT] [*FV*]

[+/-] [–] 10,000 [=]

Solution is **2,133.19**.

Solution 5

E Chapter 13, Dollar-weighted Rate of Return

The income is the withdrawals minus the deposits, treating the initial balance as a deposit and the final balance as a withdrawal:

$$\text{Income}=\text{Withdrawals}-\text{Deposits}=60+5+25+80+35-75-10\times 12=10$$

The fund exposure is the average amount in the fund:

$$\text{Fund exposure}=\sum(\text{Net deposit})(\text{Time deposit is in the fund})$$

$$=75+10\left(\frac{11}{12}+\frac{10}{12}+\cdots+\frac{1}{12}+\frac{0}{12}\right)-5\times\frac{10}{12}-25\times\frac{6}{12}-80\times\frac{2.5}{12}-35\times\frac{2}{12}$$

$$=75+10\left(\frac{11\times 12}{2\times 12}\right)-\frac{470}{12}=90.8333$$

The simple interest approximation for the dollar-weighted rate of return is the income divided by the fund exposure:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{10}{90.8333} = \mathbf{0.1101}$$

Solution 6

C Chapter 8, Varying Annuities

The equation of value at the end of 1 year can be used to solve for $a_{\overline{n|}}$:

$$77.1 \times 1.105 = (Ia)_{\overline{n|}} + \frac{n}{i}v^n$$

$$77.1 \times 1.105 \times i = \ddot{a}_{\overline{n|}} - nv^n + nv^n$$

$$77.1 \times 0.105 = \frac{\ddot{a}_{\overline{n|}}}{1.105}$$

$$a_{\overline{n|}} = 8.0955$$

The BA-II Plus can be used to solve for n:

10.5 [I/Y] -8.0955 [PV] 1 [PMT] [CPT] [N]

Result is **19.00**.

Solution 7

C Chapter 8, Varying Annuities

The accumulated value is:

$$\left[1{,}000(0.06)+100\right]1.09^9 + \left[900(0.06)+100\right]1.09^8 + \cdots + \left[100(0.06)+100\right]$$

$$= 160 \times 1.09^9 + 154 \times 1.09^8 + \cdots + 106$$

Using the PIn method, we have:

$P_1 = 160$ $\quad I = -6$ $\quad n = 10$

The present value is:

$$PV_0 = \left(P_1 + \frac{I}{i}\right)a_{\overline{n|}} - \frac{In}{i}v^n = \left(160 + \frac{-6}{0.09}\right) \times \frac{1-(1.09)^{-10}}{0.09} - \frac{-6 \times 10}{0.09}(1.09)^{-10}$$

$$= 93.3333 \times 6.4177 + 281.6072 = 880.5886$$

The accumulated value at the end of 10 years is:

$$880.5886 \times 1.09^{10} = \mathbf{2{,}084.67}$$

Using the BA II Plus, we have:

10 [N] 9 [I/Y] 160 [–] 6 [÷] 0.09 [=] [PMT]

6 [×] 10 [÷] 0.09 [=] [FV]

[CPT] [PV]

$PV = -880.5886$

0 [*PMT*]

[CPT] [*FV*] Answer = **2,084.67**

Question 8 has been deleted from the set of sample questions.

Solution 9

D Chapter 12, Loans

The first 10 payments pay the principal down at a rate that is equal to 50% of the interest rate. Since the interest rate is 10%, the portion of the principal that is paid down by each of the first 10 payments is:

$$(1.50 - 1.00) \times 0.10 = 0.05$$

After 10 years, the original principal has been reduced by 5% for 10 years. The equation of value at the end of 10 years is:

$$1{,}000 \times 0.95^{10} = Xa_{\overline{10}|0.10}$$

$$598.7469 = X \times \frac{1 - 1.10^{-10}}{0.10}$$

$$X = \mathbf{97.44}$$

The BA-II Plus can be used to answer this question:

1,000 [×] 0.95 [y^x] 10 [=] [*PV*]

10 [*N*] 10 [*I/Y*] [CPT] [*PMT*]

Result is –97.4417. Solution is **97.44**.

Solution 10

B Chapter 15, Bonds

The book value at the end of 6 years is:

$$BV_6 = 0.08 \times 10{,}000 \times a_{\overline{4}|0.06} + \frac{10{,}000}{1.06^4} = 800 \times 3.4651 + 7{,}920.9366$$

$$= 10{,}693.0211$$

The interest (which is also known as the investment income) portion of the 7th payment is:

$$InvInc_7 = BV_6 \times y = 10{,}693.0211 \times 0.06 = \mathbf{641.58}$$

The BA-II Plus can be used to answer this question:

4 [*N*] 6 [*I/Y*] 0.08 [×] 10,000 [=] [*PMT*] 10,000 [*FV*]

[CPT] [*PV*]

Result is –10,693.0211.

[×] 0.06 [=]

Result is –641.5813. Answer is **641.58**.

Solution 11

A Chapter 11, Geometric Varying Annuities

At the end of 5 years, the value of the perpetuity's remaining payments is equal to the value of the 25-year annuity-immediate:

$$\frac{100}{0.08} = X\left(v + 1.08v^2 + 1.08^2v^3 + \cdots + 1.08^{24}v^{25}\right)$$

$$\frac{100}{0.08} = X\left(\frac{1}{1.08} + 1.08\left(\frac{1}{1.08}\right)^2 + 1.08^2\left(\frac{1}{1.08}\right)^3 + \cdots + 1.08^{24}\left(\frac{1}{1.08}\right)^{25}\right)$$

$$\frac{100}{0.08} = X\left(\frac{25}{1.08}\right)$$

$$X = \frac{100}{0.08} \times \frac{1.08}{25}$$

$$X = \mathbf{54}$$

Solution 12

C Chapter 5, Accumulated Value

The \$10 initial deposit accumulates over the first 10 years (40 quarters) at a nominal discount rate of d compounded quarterly and then over the next 20 years (40 half years) at a nominal interest rate of 6% compounded semiannually. The \$20 deposit at time 15 years accumulates for 15 years (30 half years) at a nominal interest rate of 6% compounded semiannually.

The equation of value at the end of 30 years can be solved for d:

$$\frac{10}{\left(1 - \frac{d}{4}\right)^{40}}(1.03)^{40} + 20(1.03)^{30} = 100$$

$$d = \mathbf{0.0453}$$

Solution 13

E Chapter 5, Varying Force of Interest

At time 3, before the deposit of X, the value of the fund is:

$$100e^{\int_0^3 \frac{t^2}{100}dt} = 100e^{\left.\frac{t^3}{300}\right|_0^3} = 100e^{\frac{27}{300}}$$

A deposit of X is made at time 3, and the interest earned from time 3 to time 6 is equal to X:

$$\left(100e^{\frac{27}{300}}+X\right)\left[e^{\left(\frac{6^3}{300}-\frac{3^3}{300}\right)}-1\right]=X$$

$$\left(100e^{\frac{27}{300}}+X\right)\left[e^{0.63}-1\right]=X$$

$$96.0259=0.1224X$$

$$X=\mathbf{784.59}$$

Solution 14

A Chapter 11, Geometric Progression Annuities

Problems involving geometric annuities can be solved using the formula for a geometric series:

$$S_n=a+ar+ar^2+\cdots+ar^{n-1}=\frac{a(1-r^n)}{1-r}$$

$$=\frac{\text{First term}-\text{Term that would come next}}{1-\text{Ratio}}$$

The present value of the perpetuity-immediate is 167.50:

$$10v+10v^2+10v^3+10v^4+10v^5+10\left[(1+K)v^6+(1+K)^2v^7+\cdots\right]=167.50$$

$$10a_{\overline{5|}}+10\times\frac{(1+K)v^6-0}{1-(1+K)v}=167.50$$

$$10\times\frac{1-1.092^{-5}}{0.092}+10\times\frac{(1+K)v^6}{1-(1+K)v}=167.50$$

$$10\times3.8696+10\times\frac{(1+K)v^6}{1-(1+K)v}=167.50$$

$$\frac{1}{\frac{1}{(1+K)}-v}=21.8407$$

$$K=0.0400$$

The question referred to $K\%$ instead of K, so we multiply the value of K found above by 100:

$$0.0400\times100=\mathbf{4.00}$$

Solution 15

B Chapter 12, Loans

The amount of the equal annual payments under option (i) is:

$$\frac{2{,}000}{a_{\overline{10|}0.0807}}=\frac{2{,}000}{6.6889}=2{,}990.0073$$

Alternatively, the amount of the equal annual payments under option (i) can be found using the BA-II Plus calculator:

10 [*N*] 8.07 [*I*/*Y*] 2,000 [+/-][*PV*] [CPT] [*PMT*]

Result is 299.0007.

The sum of the payments under option (i) is:

$$299.0007 \times 10 = 2{,}990.0073$$

Since the payments of 200 under option (ii) are over and above the payment of the interest, the balance of the loan decreases by 200 year. Therefore, the interest payments decline each year. The sum of the payments under option (ii) is:

$$200 \times 10 + i(2{,}000 + 1{,}800 + \cdots + 200) = 2{,}000 + 200i(10 + 9 + \cdots + 1)$$

$$= 2{,}000 + 200i \times \frac{10 \times 11}{2} = 2{,}000 + 11{,}000i$$

Setting the sum of the payments under option (i) equal to the sum of the payments under option (ii) allows us to solve for i:

$$2{,}990.0073 = 2{,}000 + 11{,}000i$$

$$i = \frac{990.0073}{11{,}000}$$

$$i = \mathbf{0.0900}$$

Solution 16

B Chapter 11, Geometric Progression Annuities

There are 60 monthly payments. We use one month as the unit of time:

$$\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075$$

$$v = 1.0075^{-1} = 0.99256$$

The 60 payments are described below:

Time	Payment
1	1,000
2	$1{,}000 \times 0.98$
3	$1{,}000 \times 0.98^{2}$
…	…
40	$1{,}000 \times 0.98^{39}$
41	$1{,}000 \times 0.98^{40}$
…	…
60	$1{,}000 \times 0.98^{59}$

The outstanding balance after the 40th payment is the present value of the payments after the 40th payment:

$$PV_{40} = 1,000 \times 0.98^{40} v + 1,000 \times 0.98^{41} v^2 + \cdots + 1,000 \times 0.98^{59} v^{20}$$
$$= 1,000 \times \frac{0.98^{40} v - 0.98^{60} v^{21}}{1 - 0.98v} = \mathbf{6,889.11}$$

Solution 17

C Chapter 6, Level Annuities

The equation of value at the end of $3n$ years can be used to find i:

$$8,000 = 98 s_{\overline{n|}} (1+i)^{2n} + 196 s_{\overline{2n|}}$$
$$8,000i = 98\left[(1+i)^n - 1\right](1+i)^{2n} + 196\left[(1+i)^{2n} - 1\right]$$
$$8,000i = 98\left[2-1\right]4 + 196\left[4-1\right]$$
$$i = \mathbf{0.1225}$$

Solution 18

B Chapter 8, Varying Annuities

We use one month as the unit of time. The monthly effective interest rate is:

$$\left(1 + \frac{0.09}{4}\right)^{1/3} - 1 = 0.00744$$

Using the PIn method, we have:

$P_1 = 2$ $\quad I = 2$ $\quad n = 60$ $\quad i = 0.00744$

The present value is:

$$PV_0 = \left(P_1 + \frac{I}{i}\right) a_{\overline{n|}} - \frac{In}{i} v^n$$
$$= \left(2 + \frac{2}{0.00744}\right) \times \frac{1 - (1.00744)^{-60}}{0.00744} - \frac{2 \times 60}{0.00744}(1.00744)^{-60}$$
$$= 270.6568 \times 48.2485 - 10,329.5813 = \mathbf{2,729.21}$$

Using the BA II Plus, we have:

0.09 [÷] 4 [+] 1 [=] [y^x] 3 [1/*x*] [=] [–] 1 [=] [STO] 1

60 [*N*] [RCL] 1 [×] 100 [=] [*I*/*Y*] 2 [+] 2 [÷] [RCL] 1 [=] [*PMT*]

2 [×] 60 [÷] [RCL] 1 [=] [+/-] [*FV*]

[CPT] [*PV*]

Result is *PV* = –2,728.11. Answer is **2,729.21**

Solution 19

C Chapter 13, Dollar-Weighted and Time-Weighted Return

We can use the simple interest approximation to find the dollar-weighted rate of return for account K. The income and fund exposure are:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = 125 + X - 100 - 2X = 25 - X$$

$$\text{Fund exposure} = \sum(\text{Net deposit})(\text{Time deposit is in the fund})$$
$$= 100 - 0.5X + 0.25(2X) = 100$$

The dollar-weighted rate of return is approximately:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{25 - X}{100}$$

We set the dollar-weighted return for account K equal to the time-weighted return for account L:

$$\frac{25 - X}{100} = \frac{125}{100} \times \frac{105.8}{125 - X} - 1$$
$$25 - X = \frac{125 \times 105.8}{125 - X} - 100$$
$$125 - X = \frac{13,225}{125 - X}$$
$$(125 - X)^2 = 13,225$$
$$125 - X = \pm 115$$
$$X = 10 \quad \text{or} \quad X = 240$$

There are two possibilities for the value of i:

$$\frac{25 - X}{100} = \frac{25 - 10}{100} = 0.15 \quad \text{or} \quad \frac{25 - 240}{100} = \frac{-215}{100} = -2.15$$

We use the positive value of **15%**.

Solution 20

A Chapter 3, Present Value

The equation of value at the outset is:

$$100 + 200v^n + 300v^{2n} = 600v^{10}$$
$$100 + 200 \times 0.76 + 300 \times 0.76^2 = 600v^{10}$$
$$0.7088 = v^{10}$$
$$i = \mathbf{0.03502}$$

Solution 21

A Chapter 10, Continuously Payable Annuities

The equation of value at time 10 is:

$$AV_b = \int_a^b Pmt_t e^{\int_t^b r_s ds} dt$$

$$20{,}000 = \int_0^{10} k(8+t)e^{\int_t^{10}(8+s)^{-1}ds}dt$$

We begin by evaluating the integral in the exponent:

$$\int_t^{10}(8+s)^{-1}ds = \ln(8+s)\Big|_t^{10} = \ln(18) - \ln(8+t) = \ln\left(\frac{18}{8+t}\right)$$

The equation of value can now be used to solve for k:

$$20{,}000 = \int_0^{10} k(8+t)\left(\frac{18}{8+t}\right)dt$$

$$20{,}000 = \int_0^{10} 18k\,dt$$

$$20{,}000 = k(180-0)$$

$$k = \mathbf{111.11}$$

Solution 22

D Chapter 15, Bonds

The price of the bond is:

$$P = Coup \times a_{\overline{2n}|i} + Rv^{2n} = 1{,}000r \times \frac{1-v^{2n}}{i} + 381.50$$

$$= 1{,}000 \times 1.03125(1-v^{2n}) + 381.50 = 1{,}031.25(1-0.5889^2) + 381.50$$

$$= \mathbf{1{,}055.11}$$

Solution 23

D Chapter 13, Net Present Value

The net present value of Project P is:

$$NPV_0 = PV_0(\text{Cash Inflows}) - PV_0(\text{Cash Outflows}) = -4{,}000 + \frac{2{,}000}{1.10} + \frac{4{,}000}{1.10^2}$$

$$= 1{,}123.9669$$

We can set this equal to the next present value of Project Q:

$$1{,}123.9669 = 2{,}000 + \frac{4{,}000}{1.10} - \frac{X}{1.10^2}$$

$$X = \mathbf{5{,}460}$$

Solution 24

E Chapter 12, Sinking Fund

We can use the BA-II Plus calculator to find the amount of the annual payment:

20 [*N*] 6.5 [*I*/*Y*] 20,000 [*PV*] [CPT] [*PMT*]

Result is –1,815.1279.

We reduce the payment amount by the interest that is paid to the lender in order to obtain the sinking fund payment that must accumulate to 20,000 at the end of 20 years. We can use the calculator to find the interest rate that the sinking fund must earn. Continuing from the calculation above, the keystrokes are:

[+] 0.08 [×] 20,000 [=] [*PMT*]

Result is –215.1279, indicating that the sinking fund payment is 215.1279.

0 [*PV*] 20,000 [*FV*] [CPT] [*I*/*Y*]

Result is 14.1792.

The solution is **14.18%**.

Solution 25

D Chapter 6, Perpetuities

Brian's share of the present value of the perpetuity is 40%:

$$Xa_{\overline{n|}} = 0.4 \times \frac{X}{i}$$

$$1 - v^n = 0.4$$

$$v^n = 0.6$$

The charity's share of the present value of the perpetuity is *K*:

$$\frac{X}{i} \times v^{2n} = K \times \frac{X}{i}$$

$$0.6^2 = K$$

$$K = \mathbf{0.36}$$

Solution 26

D Chapter 12, Loans

The interest paid by Seth is:

$$5{,}000 \times \left(1.06^{10} - 1\right) = 3{,}954.2385$$

The interest paid by Janice is:

$$5{,}000 \times 10 \times 0.06 = 3{,}000$$

The interest paid by Lori is the sum of the 10 payments minus the total principal paid. The total principal paid is equal to the initial principal of 5,000:

$$\frac{5,000}{a_{\overline{10}|0.06}}\times 10-5,000=\frac{5,000}{7.3601}\times 10-5,000=679.3398\times 10-5,000$$
$$=1,793.3979$$

The total amount of interest paid on all 3 loans is:

$$3,954.2385+3,000+1,793.3979=\mathbf{8,747.64}$$

Solution 27

E Chapter 3, Accumulated Value

Since the amount of interest earned in Bruce's account during the 11th year is equal to the amount of interest earned in Robbie's account during the 17th year, the amount in Bruce's account at the end of the 10th year must be equal to the amount in Robbie's account at the end of the 16th year:

$$100(1+i)^{10}=50(1+i)^{16}$$
$$2=(1+i)^{6}$$
$$i=0.1225$$

The interest earned in Bruce's account in the 11th year is:

$$X=100(1+i)^{10}i=100(1.1225)^{10}\times 0.1225=\mathbf{38.88}$$

Solution 28

D Chapter 12, Loans

The outstanding principal at the end of $(t-1)$ years is:

$$a_{\overline{n-(t-1)}|}=a_{\overline{n-t+1}|}$$

The outstanding principal at the end of t years is:

$$a_{\overline{n-t}|}$$

The sum of the interest paid in year t and the principal paid in year $(t+1)$ is:

$$X=a_{\overline{n-t+1}|}\times i+(1-a_{\overline{n-t}|}\times i)=1-v^{n-t+1}+\left[1-(1-v^{n-t})\right]=1-v^{n-t+1}+v^{n-t}$$
$$=1-v^{n-t}(v-1)=1+v^{n-t}d$$

The answer is **Choice D**.

Solution 29

B Chapter 7, Perpetuities

The present value of the first annuity can be used to find the effective 3-year interest rate:

$$\frac{10}{\frac{i^{(1/3)}}{1/3}} = 32$$

$$\frac{i^{(1/3)}}{1/3} = 0.3125$$

The 4-month effective interest rate is found below:

$$\left(1+\frac{i^{(m)}}{m}\right)^m = \left(1+\frac{i^{(p)}}{p}\right)^p$$

$$(1.3125)^{1/3} = \left(1+\frac{i^{(3)}}{3}\right)^3$$

$$\frac{i^{(3)}}{3} = \left(1.3125^{1/3}\right)^{1/3} - 1 = 0.03068$$

The present value of the second perpetuity is:

$$X = \frac{1}{0.03068} = \mathbf{32.60}$$

Solution 30

D Chapter 17, Asset-Liability Matching

On 12/31/2017, the company will receive the par value of 822,703, leaving the following net liability:

$$1{,}000{,}000 - 822{,}703 = 177{,}297$$

Under Scenario A, the profit is:

$$0.05 \times 822{,}703 s_{\overline{4}|0.045} - 177{,}297 = 0.05 \times 822{,}703 \times \frac{1.045^4 - 1}{0.045} - 177{,}297$$
$$= -1{,}312.97$$

Under Scenario B, the profit is:

$$0.05 \times 822{,}703 s_{\overline{4}|0.055} - 177{,}297 = 0.05 \times 822{,}703 \times \frac{1.055^4 - 1}{0.055} - 177{,}297$$
$$= 1{,}322.78$$

Choice D best describes the insurance company's profit.

Solution 31

D Chapter 11, Geometric Progression Annuities

The 20 payments are described below:

Time	Payment
1	$5,000 \times 1.07$
2	$5,000 \times 1.07^2$
3	$5,000 \times 1.07^3$
...	...
20	$5,000 \times 1.07^{20}$

The present value of the payments is:

$$5,000 \times 1.07v + 5,000 \times 1.07^2 v^2 + \cdots + 5,000 \times 1.07^{20} v^{20}$$

$$= 5,000 \times \frac{\frac{1.07}{1.05} - \left(\frac{1.07}{1.05}\right)^{21}}{1 - \frac{1.07}{1.05}} = \mathbf{122,633.60}$$

Solution 32

C Chapter 13, Net Present Value

The 60,000 received at the end of 3 years is lent for 1 year at 4%. The net present value is:

$$NPV = \frac{60,000(1.04) + 60,000}{1.05^4} - 100,000 = \mathbf{698.78}$$

Solution 33

B Chapter 18, Spot Rates

The value of the bond is:

$$\frac{60}{1.07} + \frac{60}{1.08^2} + \frac{1,060}{1.09^3} = \mathbf{926.03}$$

Solution 34

E Chapter 18, Spot Rates

We can use the BA-II Plus calculator to answer this question:

60 [÷] 1.07 [+] 60 [÷] 1.08 [x^2] [+] 1,060 [÷] 1.09 [y^x] 3 [=]

(Result is 926.0296)

[+/-] [PV] 3 [N] 60 [PMT] 1,000 [FV]

[CPT] [I/Y]

(Result is 8.9180)

Answer is **8.92%**.

Solution 35

C Chapter 16, Duration

The modified duration can be expressed in terms of the derivative of the bond's price or in terms of the Macaulay duration:

$$ModDur = -\frac{P'\left(y^{(m)}\right)}{P\left(y^{(m)}\right)} = \frac{MacDur}{1+\frac{y^{(m)}}{m}}$$

$$-\frac{-700}{100} = \frac{MacDur}{1+0.08}$$

$$MacDur = \mathbf{7.56}$$

Solution 36

C Chapter 16, Duration

The price of the stock and the derivative of its price are:

$$P(y) = \frac{Div}{y} = Div \times y^{-1}$$

$$P'(y) = -Div \times y^{-2}$$

The modified duration is:

$$ModDur = -\frac{P'(y)}{P(y)} = -\frac{-Div \times y^{-2}}{Div \times y^{-1}} = \frac{1}{y} = \frac{1}{0.10} = 10$$

The Macaulay duration is:

$$MacDur = ModDur \times (1+y) = 10 \times 1.10 = \mathbf{11}$$

Solution 37

B Chapter 16, Duration

The price of the stock and the derivative of its price are:

$$P(y) = \frac{Div}{y-g} = Div \times (y-g)^{-1}$$

$$P'(y) = -Div \times (y-g)^{-2}$$

The modified duration is:

$$ModDur = -\frac{P'(y)}{P(y)} = -\frac{-Div \times (y-g)^{-2}}{Div \times (y-g)^{-1}} = \frac{1}{y-g} = \frac{1}{0.05-0.02} = \frac{1}{0.03}$$

$$= 33.3333$$

The Macaulay duration is:

$$MacDur = ModDur \times (1+y) = 33.3333 \times 1.05 = \mathbf{35}$$

Questions 38-44 have been deleted from the set of sample questions.

Solution 45

A Chapter 13, Project Appraisal

We can use the fact that the time-weighted rate of return is zero to solve for X:

$$(1+0) = \frac{12}{10} \times \frac{X}{12+X}$$
$$\frac{10}{12} = \frac{X}{12+X}$$
$$120 + 10X = 12X$$
$$X = 60$$

The investment income and the exposure of the fund to interest are:

$$\text{Income} = \text{Withdrawals} - \text{Deposits} = X - 10 - X = -10$$
$$\text{Fund exposure} = \sum (\text{Net deposit})(\text{Time deposit is in the fund}) = 10 + 0.5X$$
$$= 10 + 0.5 \times 60 = 40$$

The simple interest approximation for the dollar-weighted rate of return is:

$$i = \frac{\text{Income}}{\text{Fund exposure}} = \frac{-10}{40} = \mathbf{-0.25}$$

Solution 46

A Chapter 12, Loans

The final payment is the loan balance at the end of year 3 accrued with interest:

$$Pmt = 559.12 \times 1.08 = 603.8496$$

The initial loan balance is:

$$Pmt \times a_{\overline{4|}} = 603.8496 \times \frac{1-1.08^{-4}}{0.08} = 2{,}000.0265$$

The first principal payment is equal to the level payment minus the interest on the initial loan balance:

$$603.8496 - 2{,}000.0265 \times 0.08 = \mathbf{443.8475}$$

Solution 47

B Chapter 15, Bonds

Since the yield is equal to the coupon rate, the price of the bond is 1,000. Since the investment in the bond results in a yield of 7%, we have the following equation of value:

$$1,000(1.07)^{10} = 30s_{\overline{20}|(1+i)^{0.5}-1} + 1,000$$

We can use the BA-II Plus calculator to answer this question:

1,000 [×] 1.07 [y^x] 10 [–] 1,000 [=] [*FV*]

20 [*N*] 30 [+/-] [*PMT*] [CPT] [*I*/*Y*]

Result is 4.7597.

[÷] 100 + 1 [=] [x^2] [–] 1 [=]

Solution is **0.09746**.

Solution 48

A Chapter 7, Level Annuities

The monthly effective interest rate is:

$$\frac{0.08}{12} = 0.00667$$

To have 3,000 of monthly income beginning on his 65th birthday, the man needs the following lump sum on his 65th birthday:

$$\frac{3,000}{9.65} \times 1,000 = 310,880.83$$

His contributions must accumulate to 310,880.83:

$$X\ddot{s}_{\overline{25\times 12}|0.00667} = 310,880.83$$

$$X \times \frac{1.00667^{300} - 1}{0.00667} \times 1.00667 = 310,880.83$$

$$957.3666X = 310,880.83$$

$$X = \mathbf{324.72}$$

We can use the BA II Plus to answer this question:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

–3,000 [÷] 9.65 [×] 1,000 [=] [*FV*]

25 [×] 12 [=] [*N*] 8 [÷] 12 [=] [*I*/*Y*]

[CPT] [*PMT*]

Answer is **324.72**

Solution 49

D Chapter 6, Level Annuities

The parents make 17 contributions of *X*. On the daughter's 18th birthday, the equation of value is:

$$X\left[1.05^{17}+1.05^{16}+\cdots+1.05\right]=50,000\left[1+v+v^2+v^3\right]$$

$$X\sum_{k=1}^{17}1.05^{k}=50,000\left[1+v+v^2+v^3\right]$$

Only Choices A and D show 17 contributions, so the correct answer must be Choice A or Choice D.

The right side of the equation in Choice A shows the value of the withdrawals at time 17, but the left side of Choice A shows the value of the contributions at time 0, so the equation of value is not correct.

The right side of Choice D shows the value of the withdrawals at time 18, and the left side of Choice D shows the value of the contributions at time 18, so the equation of value is valid, and **Choice D** is the correct answer.

Question 50 has been deleted from the set of sample questions.

Solution 51

D Chapter 17, Dedication

The quantity of Bond II to purchase is the quantity that produces a cash flow of \$1,000 at time 1:

$$Q_{II}=\frac{1,000}{1,025}=0.97561$$

The quantity of Bond I to purchase is the quantity that produces the net liability remaining after the payment from Bond II is received:

$$Q_{I}=\frac{1,000-25\times 0.97561}{1,040}=0.93809$$

Choice D is the correct answer.

Solution 52

C Chapter 17, Dedication

The liability of 1,000 due in two years is met by arranging a payment of 1,000 in two years from Mortgage II. Since Mortgage II makes an equal-sized payment at time 1, it also pays 1,000 at time 1. That leaves 1,000 of net liability at time 1 to be met by Mortgage I. The amount lent is:

$$X+Y=\frac{1,000}{1.06}+\frac{1,000}{1.07}+\frac{1,000}{1.07^2}=\mathbf{2,751.41}$$

Solution 53

A Chapter 18, Forward Rates

Bond II must produce a cash flow at time 2 that is sufficient to grow to 2,000 at a 6.5% interest rate:

$$\frac{2,000}{1.065} = 1,877.9343$$

The combined prices of Bond I and Bond II are:

$$\frac{1,000}{1.06} + \frac{1,877.9343}{1.07^2} = \mathbf{2,583.66}$$

Solution 54

C Chapter 15, Callable Bonds

Since the coupon rate is greater than the yield-to-worst, the bond is a premium bond. The yield-to-worst can therefore be found by identifying each interval defined by level redemption prices and considering the possibility that the bond is called at the beginning of each interval. In this case, there is only one such interval, and it runs from time 15 years until maturity. The yield-to-worst is based on the beginning of the interval, which occurs at the time 15 years. The equation of value is:

$$1,722.25 = 0.04X a_{\overline{30}|0.03} + \frac{X}{1.03^{30}}$$

We can use the BA II Plus to answer this question:

30 [*N*] 3 [*I/Y*] 0.04 [*PMT*] 1 [*FV*]

[CPT] [*PV*]

Result is –1.1960.

[1/*x*] [×] 1,722.25 [=]

Result is –1,440.0030. Answer is **1,440.00**.

Solution 55

B Chapter 15, Callable Bonds

We observe that the coupon of 40 is greater than the yield-to-worst times the final redemption value:

$$YTW \times R = 0.03 \times 1,100 = 33$$

Therefore, the bond is a premium bond, and the earliest possible redemption within each interval of level redemption prices should be considered. The price-to-worst is the minimum of the two resulting prices:

$$Min\left[40 \times a_{\overline{30}|0.03} + \frac{1,200}{1.03^{30}},\ 40 \times a_{\overline{40}|0.03} + \frac{1,100}{1.03^{40}}\right]$$

We can use the BA II Plus to answer this question:

30 [*N*] 3 [*I*/*Y*] 40 [*PMT*] 1,200 [*FV*]

[CPT] [*PV*]

Result is –1,278.4018.

[STO] 1

40 [*N*] 1,100 [*FV*] [CPT] [*PV*]

Result is –1,261.8034.

Since 1,261.8034 is less than 1,278.4018, the price-to-worst is **1,261.8034**.

Solution 56

E Chapter 15, Callable Bonds

The coupon is less than the product of the yield-to-worst and the final redemption value, so the bond is a discount bond:

$$0.02 \times X < 0.03 \times X \quad \Rightarrow \quad Coup < YTW \times R_{t_k}$$

Therefore, the yield-to-worst is based on the latest possible redemption, which occurs at the end of 10 years. The equation of value is:

$$1,021.50 = 0.02Xa_{\overline{20}|0.03} + \frac{X}{1.03^{20}}$$

We can use the BA II Plus to answer this question:

20 [*N*] 3 [*I*/*Y*] 0.02 [*PMT*] 1 [*FV*]

[CPT] [*PV*]

Result is –0.8512.

[1/*x*] [×] 1,021.50 [=]

Result is –1,200.0349. Answer is **1,200.03**.

Solution 57

B Chapter 15, Callable Bonds

Since the price is less than the redemption value of 1,100, the bond is a discount bond. Therefore, the yield-to-worst is based on the latest possible redemption, which occurs at the end of 10 years. The equation of value is:

$$1,021.50 = (0.02 \times 1,100)a_{\overline{20}|y} + \frac{1,100}{(1+y)^{20}}$$

We can use the BA II Plus to answer this question:

20 [*N*] 1,021.50 [+/-] [*PV*] 0.02 [×] 1,100 [=] [*PMT*] 1,100 [*FV*]

[CPT] [*I*/*Y*]

Result is 2.4559.

[×] 2 [=]

Result is 4.9117.

Answer is **4.91%**.

Question 58 has been deleted from the set of sample questions.

Solution 59

C Chapter 17, Asset-Liability Management

The Macaulay duration of the liability is:

$$MacDur = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{15,000(Ia)_{\overline{15|}}}{15,000a_{\overline{15|}}} = \frac{\frac{\ddot{a}_{\overline{15|}} - 15v^{15}}{0.062}}{9.5866} = \frac{66.0721}{9.5866}$$

$$= 6.8921$$

Let X be the percentage of the present value that is invested in the 5-year bonds:

$$6.8921 = 5X + (1 - X)10$$

$$X = 0.6216$$

The amount invested in the 5-year bonds is:

$$X \times 35,000 \times a_{\overline{15|}} = 0.6216 \times 35,000 \times 9.5866 = \mathbf{208,556.21}$$

The BA II Plus can be used to answer this question:

15 [N] 6.2 [I/Y] 1[PMT] [CPT] [PV] [+/-]

Result is 9.5865.

[STO] 1 [×] 1.062 [−] 15 [÷] 1.062 [y^x] 15 [=] [÷] 0.062 [=]

Result is 66.0721.

[÷] [RCL] 1 [=]

Result is 6.8921.

[−] 10 [=] [÷] 5 [+/-] [=]

Result is 0.6216.

[×] [RCL] 1 [×] 35,000 [=]

Solution is **208,556.21**.

Solution 60

A Chapter 11, Geometric Progression Annuities

The 16 payments are described below:

Time	Payment
1	$2,000$
2	$2,000 \times 1.03$
3	$2,000 \times 1.03^2$
…	…
8	$2,000 \times 1.03^7$
9	$2,000 \times 1.03^7 \times 0.97$
10	$2,000 \times 1.03^7 \times 0.97^2$
…	…
16	$2,000 \times 1.03^7 \times 0.97^8$

The present value of the payments is:

$$L = 2,000v + 2,000v^2 \times 1.03 + \cdots + 2,000v^8 \times 1.03^7$$
$$+2,000 \times 1.03^7 v^8 \left[0.97v + 0.97^2 v^2 + \cdots + 0.97^8 v^8\right]$$
$$= 2,000 \times \frac{v - 1.03^8 v^9}{1 - 1.03v} + 2,000 \times 1.03^7 v^8 \times \frac{0.97v - 0.97^9 v^9}{1 - 0.97v}$$
$$= 2,000 \times 6.5682 + 1,431.5956 \times 5.2754$$
$$= 13,136.4140 + 7,552.2193$$
$$= \mathbf{20,688.63}$$

Solution 61

E Chapter 5, Varying Force of Interest

The equation of value is:

$$500e^{\int_0^t \delta_s ds} = 2,000$$
$$\exp\left[\int_0^t \frac{\frac{s^2}{100}}{3+\frac{s^3}{150}} ds\right] = 4$$
$$\int_0^t \frac{\frac{s^2}{100}}{3+\frac{s^3}{150}} ds = \ln 4$$

To evaluate the integral, let’s use the following substitution:

$$u = 3 + \frac{s^3}{150}$$
$$du = \frac{s^2}{50} ds$$

The integral is:

$$\int_0^t \frac{\frac{s^2}{100}}{3+\frac{s^3}{150}} ds = \int_{s=0}^{s=t} 0.5u^{-1} du = 0.5 \ln u \Big|_{s=0}^{s=t} = 0.5 \ln\left(3 + \frac{s^3}{150}\right)\Bigg|_{s=0}^{s=t}$$

$$= 0.5\left[\ln\left(3 + \frac{t^3}{150}\right) - \ln(3)\right] = \ln\left(\sqrt{\frac{3 + \frac{t^3}{150}}{3}}\right)$$

We can now solve for t:

$$\sqrt{\frac{3 + \frac{t^3}{150}}{3}} = 4$$

$$t = \mathbf{18.9}$$

Solution 62

E Chapter 15, Bonds

The rate of growth of the accumulation of discount is equal to the yield:

$$\frac{DA_{t+k}}{DA_t} = (1+y)^k$$

$$\frac{306.69}{194.82} = (1+y)^5$$

$$y = 0.09500$$

We use the discount accumulated in the 15th year to find the difference between the yield times the redemption value and the coupon:

$$DA_t = (Ry - Coup)v^{n-t+1}$$

$$194.82 = \frac{Ry - Coup}{1.09500^{40-15+1}}$$

$$Ry - Coup = 2{,}062.4201$$

The discount at the time of purchase is:

$$\text{Discount} = (Ry - Coup)a_{\overline{n}|y} = 2{,}062.4201 \times a_{\overline{40}|0.09500}$$

$$= 2{,}062.4201 \times 10.2475 = \mathbf{21{,}134.59}$$

Solution 63

A Chapter 12, Loans

We begin by finding the level payment amount:

$$Prn_t = v^{n-t+1} \times Pmt$$

$$699.68 = \frac{Pmt}{1.0475^{8-5+1}}$$

$$Pmt = 842.3946$$

The total amount of interest paid on the loan is equal to the total amount of the payments minus the initial loan balance:

$$8 \times 842.3946 - 842.3946 \times a_{\overline{8|}} = 6{,}739.1570 - 842.3946 \times \frac{1 - 1.0475^{-8}}{0.0475}$$
$$= 6{,}739.1570 - 842.3946 \times 6.5290 = 6{,}739.1570 - 5{,}500.0251$$
$$= \mathbf{1{,}239.13}$$

The BA II Plus can be used to answer this question:

699.68 [×] 1.0475 [y^x] 4 [=]

Result is 842.3946.

[*PMT*] 8 [*N*] 4.75 [*I*/*Y*] [CPT] [*PV*]

Result is –5,500.0251.

[+] 8 [×] [RCL] [*PMT*] [=]

Answer is **1,239.13**.

Solution 64

D Chapter 7, Level Annuities

The monthly interest rate for the first 18 months is:

$$\frac{0.084}{12} = 0.007$$

The accumulated value of the loan after 18 months minus the accumulated value of the payments is:

$$22{,}000(1.007)^{18} - 450.30 s_{\overline{18|}0.007} = 24{,}943.2564 - 450.30 \times \frac{1.007^{18} - 1}{0.007}$$
$$= 24{,}943.2564 - 450.30 \times 19.1121 = 16{,}337.0983$$

The new interest rate is to refinance the loan is:

$$\frac{0.048}{12} = 0.004$$

The equation of value for the refinanced loan after 18 months is:

$$16{,}337.0983 = X a_{\overline{24|}0.004}$$
$$16{,}337.0983 = X \times \frac{1 - 1.004^{-24}}{0.004}$$
$$16{,}337.0983 = 22.8405X$$
$$X = \mathbf{715.27}$$

We can use the BA II Plus to answer this question:

18 [*N*] 0.7 [*I*/*Y*] 22,000 [*PV*] 450.30 [+/-] [*PMT*] [CPT] [*FV*]

(Result is –16,337.0983)

[*PV*] 24 [*N*] 0.4 [*I*/*Y*] 0 [*FV*] [CPT] [*PMT*]

Answer is **715.27**.

Solution 65

C Chapter 16, Duration

Since the bond is priced at par, its Macaulay duration is:

$$MacDur = \ddot{a}_{\overline{n}|}^{(m)} = \ddot{a}_{\overline{7}|}^{(2)} = \frac{1-1.038^{-14}}{0.076} \times 1.038 = \mathbf{5.56}$$

Solution 66

A Chapter 16, Duration

The modified duration is:

$$ModDur = \frac{MacDur}{1+\frac{y^{(m)}}{m}} = \frac{7.959}{1.072} = 7.4244$$

The estimated percentage change in the price is:

$$\%\Delta P \simeq -ModDur \times \Delta y^{(m)} = -7.4244 \times (0.08 - 0.072) = -0.05940$$

The estimate for the new price is:

$$1,000(1+\%\Delta P) = 1,000(1-0.05940) = \mathbf{940.60}$$

Solution 67

E Chapter 18, Forward Rates

The wording of this question is a little ambiguous, as it seems that we are being asked for either:

- f_2, the forward rate that applies from time 2 to time 3, or
- f_3, the forward rate that applies from time 3 to time 4

Since we are not given enough information to find f_3, we conclude that we are being asked for f_2:

$$1+f_{t-1} = \frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_2 = \frac{\frac{1}{0.85892}}{\frac{1}{0.90703}}$$

$$f_2 = \mathbf{0.05601}$$

Solution 68

C Chapter 16, Duration

Since we are not told otherwise, we assume that the yield remains constant at 5%.

After the first coupon is paid, the bond is still priced at par, so the remaining 7-year bond has a price of 5,000.

The duration of an immediate payment of 250 is 0, and the duration of the 7-year bond is d_2. A portfolio consisting of an immediate payment of 250 and the 7-year bond has a duration of d_1.

$$d_1 = MacDur_{Port} = \sum_{j=1}^{k} w_j \times MacDur_j = \frac{250}{250+5,000} \times 0 + \frac{5,000}{250+5,000} \times d_2$$
$$= 0.9524 \times d_2$$

The ratio is:

$$\frac{d_1}{d_2} = \frac{0.9524 \times d_2}{d_2} = \mathbf{0.9524}$$

Solution 69

A Chapter 17, Dedication

The quantity of Bond C that is purchased is:

$$Q_C = \frac{100}{105}$$

The quantity of Bond A that is purchased is the amount needed to cover the remaining liability after the cash flow from Bond C is received:

$$Q_A = \frac{99 - \frac{100}{105} \times 5}{107} = \mathbf{0.8807}$$

Solution 70

B Chapter 17, Redington Immunization

A is true, because unless the portfolio is cash-flow-matched, the duration of the assets can change at a different rate from the duration of the liabilities.

B is false because Redington immunization requires frequent rebalancing.

C is true. Full immunization protects against large changes in the interest rate, but Redington immunization only protects against small changes.

D is true, because Redington immunization is based on an assumption that the yield curve is flat.

E is true, because Redington immunization is based on an assumption that any shifts to the yield curve are parallel shifts.

Solution 71

D Chapter 17, Full Immunization

Since the liability is equidistant from the 2 asset cash flows, the weights (or market values) of the 2 asset cash flows must be equal. This implies that the value of *A* at time 4 is 3,000 and the value of *B* is also 3,000:

$$A(1.05)^2 = 3{,}000 \quad \Rightarrow \quad A = 2{,}721.0884$$
$$\frac{B}{1.05^2} = 3{,}000 \quad \Rightarrow \quad B = 3{,}307.5000$$

The absolute value of the difference is:

$$|A - B| = |2{,}721.0884 - 3{,}307.5000| = \mathbf{586.41}$$

Solution 72

A Chapter 17, Full Immunization

The present value of the assets is equal to the present value of the liability:

$$\frac{5{,}000}{1.03^5} + \frac{B}{1.03^{8+b}} = \frac{12{,}000}{1.03^8}$$
$$\frac{B}{1.03^{8+b}} = 5{,}159.8669$$

The duration of the assets is equal to the duration of the liability:

$$\frac{5 \times 5{,}000}{1.03^5} + \frac{(8+b)B}{1.03^{8+b}} = \frac{8 \times 12{,}000}{1.03^8}$$
$$\frac{25{,}000}{1.03^5} + (8+b) \times 5{,}159.8669 = \frac{96{,}000}{1.03^8}$$
$$8 + b = 10.5076$$
$$b = 2.5076$$

We can now solve for *B*:

$$\frac{B}{1.03^{10.5076}} = 5{,}159.8669$$
$$B = 7{,}039.2687$$

The ratio is:

$$\frac{B}{b} = \frac{7{,}039.2687}{2.5076} = \mathbf{2{,}807.12}$$

Solution 73

D Chapter 17, Immunization

The first sentence of the question could be more clearly stated as, "Trevor has asset cash flows at time 2 of A and at time 9 of B."

The duration of the asset portfolio must be equal to duration of the liability. Let X be the percentage of the asset portfolio that is invested in the asset that pays at time 2:

$$2X + 9(1 - X) = 5$$
$$-7X = -4$$
$$X = \frac{4}{7}$$

Since the present value of the asset portfolio is equal to the present value of the liability, the present values of the asset cash flows at time 0 are:

$$PV_A = \frac{4}{7} \times PV_L$$
$$PV_B = \frac{3}{7} \times PV_L$$

The amounts of the cash flows are found by accumulating their present values:

$$\frac{A}{B} = \frac{PV_A \times 1.04^2}{PV_B \times 1.04^9} = \frac{\frac{4}{7} \times PV_L \times 1.04^2}{\frac{3}{7} \times PV_L \times 1.04^9} = \frac{4}{3 \times 1.04^7} = \mathbf{1.0132}$$

Solution 74

D Chapter 15, Bonds

The price of Bond A exceeds the price of Bond B by 5,341.12:

$$10{,}000\left(\frac{i}{2} + 0.02\right)a_{\overline{20}|i/2} + \frac{10{,}000}{\left(1 + \frac{i}{2}\right)^2} - \left[10{,}000\left(\frac{i}{2} - 0.02\right)a_{\overline{20}|i/2} + \frac{10{,}000}{\left(1 + \frac{i}{2}\right)^2}\right]$$
$$= 5{,}341.12$$
$$10{,}000(0.04)a_{\overline{20}|i/2} = 5{,}341.12$$

We can use the BA II Plus to answer this question:

5,341.12 [÷] 10,000 [÷] 0.04 [=] [*PV*]

20 [*N*] 1 [+/-] [*PMT*] [CPT] [*I/Y*]

[×] 2 [=]

Result is 8.4000. Answer is **0.0840**.

Solution 75

D Chapter 12, Loans

The initial loan payment is:

$$Pmt = \frac{400{,}000}{a_{\overline{15\times 12}|0.0075}} = \frac{400{,}000}{\frac{1-1.0075^{-180}}{0.0075}} = \frac{400{,}000}{98.5934} = 4{,}057.0663$$

The balance after the 36th payment can be found using the prospective method. At the new interest rate, the smaller payments pay off the balance in 12 years:

$$4{,}057.0663 \times a_{\overline{144}|0.0075} = (4{,}057.0663 - 409.88)a_{\overline{144}|j/12}$$

$$4{,}057.0663 \times 87.8711 = 3{,}647.1863 \times a_{\overline{144}|j/12}$$

$$356{,}498.8491 = 3{,}647.1863 \times a_{\overline{144}|j/12}$$

The easiest way to find *j* is to use the BA II Plus calculator. Let's use the calculator from the beginning of this question:

180 [*N*] 9 [÷] 12 [=] [*I*/*Y*] 400,000 [+/-] [*PV*] [CPT] [*PMT*]

Result is 4,057.0663.

144 [*N*] [CPT] [*PV*]

Result is –356,498.8491.

[RCL] [*PMT*] [–] 409.88 [=] [*PMT*] [CPT] [*I*/*Y*] [×] 12 [=]

Result is 6.9000. Answer is **6.90%**.

Solution 76

D Chapter 15, Bonds

The equation of value that equates the prices of the two bonds is:

$$25 \times a_{\overline{60}|0.025} + \frac{1{,}200}{1.025^{60}} = 25 \times a_{\overline{60}|j/2} + \frac{800}{\left(1+\frac{j}{2}\right)^{60}}$$

The easiest way to find *j* is to use the BA II Plus calculator:

60 [*N*] 2.5 [*I*/*Y*] 25 [*PMT*] 1,200 [*FV*] [CPT] [*PV*]

Result is –1,045.4567.

800 [*FV*] [CPT] [*I*/*Y*] [×] 2 [=]

Result is 4.3985. Answer is **4.40%**.

Solution 77

E Chapter 3, Interest Rate Conversions

Interest is credited only at the end of each interest conversion period, so Lucas receives interest only every 6 months. Therefore, the moment at which Lucas's account is at least double the amount in Danielle's account will occur on some multiple of 6 months.

Let t be the number of 6-month periods until Lucas's account is at least double the amount in Danielle's account. We need to find the minimum integer value of t that satisfies the following:

$$\left(1+\frac{0.06}{2}\right)^t \geq 2\left(1+\frac{0.03}{12}\right)^{6t}$$
$$t\ln(1.03) \geq \ln(2) + 6t\ln(1.0025)$$
$$t \geq 47.5490$$

The smallest integer that satisfies the equation above is t = 48. The number of months in 48 6-month intervals is:

$$48 \times 6 = \mathbf{288}$$

Solution 78

B Chapter 13, Time-Weighted Rate of Return

The balance at the end of the year is:

$$5{,}000 \times 1.09 + 2{,}600 \times 1.09^{0.5} = 8{,}164.4797$$

The time-weighted rate of return is:

$$\frac{5{,}200}{5{,}000} \times \frac{8{,}164.4797}{5{,}200 + 2{,}600} - 1 = \mathbf{0.08860}$$

Solution 79

A Chapter 5, Varying Rates

The interest accumulation factors must be the same for Bill and Joe over the course of 4 years. The interest accumulation factor for Joe is:

$$\exp\left[\int_0^4 \frac{1}{K+0.25t}\,dt\right] = \exp\left[\left.\frac{\ln(K+0.25t)}{0.25}\right|_0^4\right] = \exp\left[4\times\ln(K+1) - 4\times\ln(K)\right]$$
$$= \exp\left[4\times\ln\left(\frac{K+1}{K}\right)\right] = \left(\frac{K+1}{K}\right)^4$$

The interest accumulation factor for Joe is equal to the interest accumulation factor for Bill:

$$\left(\frac{K+1}{K}\right)^4 = \left(1+\frac{K}{25}\right)^4$$
$$\frac{K+1}{K} = 1+\frac{K}{25}$$
$$K+1 = K+\frac{K^2}{25}$$
$$1 = \frac{K^2}{25}$$
$$K = 5$$

Joe's accumulated value at the end of 4 years is:

$$10\left(\frac{K+1}{K}\right)^4 = 10\left(\frac{6}{5}\right)^4 = \mathbf{20.736}$$

Solution 80

C Chapter 12, Sinking Funds

Since the student pays the interest on the loan each year, the amount needed to pay off the loan at the end of 5 years is the original amount of 1,000. Therefore, the sinking fund payments must accumulate to 1,000.

$$1{,}000 \times 2i \times s_{\overline{5}|0.8i} = 1{,}000$$
$$2{,}000i \times \frac{(1+0.8i)^5 - 1}{0.8i} = 1{,}000$$
$$(1+0.8i)^5 - 1 = 0.4$$
$$0.8i = 0.06961$$
$$i = \mathbf{0.08701}$$

Solution 81

D Chapter 12, Loans

The principal repaid in year 26 is the payment of 2,500 minus the interest on the outstanding balance at the end of 25 years:

$$X = 2{,}500 - iL_{25}$$

The interest paid in the first year is the interest rate times the initial loan balance:

$$\begin{aligned} i \times L_0 &= i\left(2{,}500 \times a_{\overline{25}|} + v^{25}L_{25}\right) = 2{,}500(1-v^{25}) + iv^{25}L_{25} \\ &= 2{,}500 - 2{,}500v^{25} + iv^{25}L_{25} = 2{,}500 - v^{25}\left(2{,}500 - iL_{25}\right) \\ &= 2{,}500 - Xv^{25} \end{aligned}$$

The final expression above matches **Choice D**.

Solution 82

A Chapter 13, Dollar-Weighted Weight of Return

The formula describes the simple interest approximation for the internal rate of return. The smaller the net deposits (i.e., the cash flows) between time 0 and time 1, relative to the initial deposit, the more exact the estimate becomes. This matches **Choice A**.

Solution 83

E Chapter 13, Time-Weighted Weight of Return

The time-weighted rate of return is based on the product of the time intervals' corresponding accumulation factors:

$$1+i=\frac{120,000}{100,000}\times\frac{130,000}{120,000+30,000}\times\frac{100,000}{130,000-50,000}$$

$$i=\mathbf{0.30}$$

Solution 84

C Chapter 6, Level Annuities

At the end of 20 years, the value in the fund is:

$$1,000\ddot{s}_{\overline{20}|0.0816}=1,000\times\frac{1.0816^{20}-1}{0.0816}\times1.0816=50,382.1558$$

The effective 6-month interest rate is:

$$1.0816^{0.5}-1=0.04$$

Let n be the number of 6-month periods that the fund can support withdrawals of 3,000:

$$3,000\ddot{a}_{\overline{n}|0.04}=50,382.1558$$

$$\frac{1-1.04^{-n}}{0.04}\times1.04=16.7941$$

$$n=26.4719$$

The 26^{th} payment of 3,000 is made at the beginning of the 26^{th} 6-month period, which is the same as the end of the 25^{th} period, so the 26^{th} payment is made at the end of 12.5 years.

Six months after the 26^{th} payment of 3,000 is made, the balance in the fund is:

$$50,382.1558(1.04)^{26}-3,000\ddot{s}_{\overline{26}|0.04}$$

$$=139,683.0046-3,000\times\frac{1.04^{26}-1}{0.04}\times1.04$$

$$=139,683.0046-3,000\times46.0842$$

$$=\mathbf{1,430.36}$$

We can use the BA II Plus to answer this question:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

20 [*N*] 8.16 [*I*/*Y*] 1,000 [*PMT*] [CPT] [*FV*]

(Result is –50,382.1558)

[*PV*] 4 [*I*/*Y*] 3,000 [*PMT*] 0 [*FV*] [CPT] [*N*]

(Result is 26.4719)

26 [*N*] [CPT] [*FV*]

Answer is **1,430.36**.

Solution 85

D Chapter 7, Perpetuities

We can use the price of the first perpetuity to find the value of i:

$$7.21 = \frac{1}{(1+i)^2 - 1} + 1$$

$$i = 0.0775$$

The annual effective interest rate used to value the second annuity is:

$$i + 0.01 = 0.0775 + 0.01 = 0.0875$$

The second perpetuity begins at the end of 1 year, so it can be valued as a perpetuity immediate accumulated for two years:

$$7.21 = \frac{R}{1.0875^3 - 1} \times 1.0875^2$$

$$R = \mathbf{1.7446}$$

Solution 86

E Chapter 8, Varying Annuities

At the end of 5 years, the balance of the loan is:

$$10,000(1.05)^5 - 100(Is)_{\overline{5|}} = 12,762.82 - 100 \times \frac{\ddot{s}_{\overline{5|}} - 5}{0.05}$$

$$= 12,762.82 - 100 \times \frac{5.8019 - 5}{0.05} = 12,762.82 - 100 \times 16.0383$$

$$= 11,158.99$$

The loan is paid off with 15 level payments of *X*:

$$Xa_{\overline{15|}} = 11{,}158.99$$

$$X \times \frac{1-v^{15}}{0.05} = 11{,}158.99$$

$$X = \mathbf{1{,}075.08}$$

We can use the BA II Plus to answer this question:

5 [*N*] 5 [*I*/*Y*] 1 [*PMT*] [CPT] [*FV*]

[+/-] [×] 1.05 [–] 5 [=] [÷] 0.05 [=] [×] 100 [+/-] [=]

[+] 10,000 [×] 1.05 [y^x] 5 [=] [*PV*]

[15] [*N*] 0 [*FV*] [CPT] [*PMT*]

Result is –1,075.08. Answer is **1,075.08.**

Solution 87

C Chapter 6, Level Annuities

The balance of the fund at the end of 10 years is:

$$\frac{5{,}000}{1.05^5} = 3{,}917.6308$$

The 10 level payments of *X* accumulate to the balance at the end of 10 years:

$$Xs_{\overline{10|}0.06} = 3{,}917.6308$$

$$X \times \frac{1.06^{10}-1}{0.06} = 3{,}917.63082$$

$$X = \mathbf{297.22}$$

We can use the BA II Plus to answer this question:

5,000 [÷] 1.05 [y^x] 5 [=] [*FV*]

10 [*N*] 6 [*I*/*Y*] [CPT] [*PMT*]

Result is –297.22. Answer is **297.22.**

Solution 88

E Chapter 12, Loans

The monthly effective rate at which the loan is originally made is:

$$\frac{0.08}{12} = 0.006667$$

The original payment amount is:

$$\frac{65{,}000}{a_{\overline{180|}0.006667}} = 621.1739$$

After the 12th payment, the remaining balance can be found using the prospective method:

$$L_{12} = 621.1739 \times a_{\overline{168|}0.006667} = 62,661.3994$$

The new payment amount is the amount needed to pay off the remaining balance at the new interest rate of 6% compounded monthly:

$$\frac{62,661.3994}{a_{\overline{180-12|}\frac{0.06}{12}}} = \frac{62,661.3994}{a_{\overline{168|}0.005}} = \mathbf{552.19}$$

Alternatively, we can use the BA II Plus to answer this question:

180 [*N*] 8 [÷] 12 [=] [*I*/*Y*] 65,000 [+/-] [*PV*] [CPT] [*PMT*]

Result is 621.1739.

168 [*N*] [CPT] [*PV*]

Result is −62,661.3994.

6 [÷] 12 [=] [*I*/*Y*] [CPT] [*PMT*]

Answer is **552.19.**

Solution 89

E Chapter 6, Level Annuities

The first tuition payment is due at the beginning of the 18th year, which is at the end of 17 years. The payment of *X* is made at the end of 18 years. The equation of value at the end of 17 years is:

$$\left[750\ddot{s}_{\overline{18|}} + X\right]v = 6,000\left[1.05^{17} + 1.05^{18}v\right]$$

$$750s_{\overline{18|}} + Xv = 6,000\left[1.05^{17} + 1.05^{18}v\right]$$

$$750 \times \frac{1.07^{18} - 1}{0.07} + \frac{X}{1.07} = 6,000 \times 4.5412$$

$$750 \times 33.9990 + \frac{X}{1.07} = 27,247.1710$$

$$25,499.2744 + \frac{X}{1.07} = 27,247.1710$$

$$X = \mathbf{1,870.25}$$

We can use the BA II Plus to answer this question:

18 [*N*] 7 [*I*/*Y*] 750 [*PMT*] [CPT] [*FV*]

1.05 [y^x] 17 [+] 1.05 [y^x] 18 [÷] 1.07 [=] [×] 6,000

[+] [RCL] [*FV*] [=] [×] 1.07 [=]

Answer is **1,870.25**.

Solution 90

B Chapter 15, Bonds

The price of the bond is:

$$P = Coup \times a_{\overline{n|}y} + Rv^n = 45 \times a_{\overline{40|}0.05} + \frac{1{,}200}{1.05^{40}} = 45 \times 17.1591 + 170.4548$$

$$= \mathbf{942.61}$$

We can use the BA II Plus to answer this question:

40 [*N*] 5 [*I*/*Y*] 45 [*PMT*] 1,200 [*FV*] [CPT] [*PV*]

Result is –942.6137. Answer is **942.61**.

Solution 91

A Chapter 15, Callable Bonds

The bond is a discount bond, because the coupon is less than the yield-to-worst times the redemption value:

$$0.05 \times 1{,}000 < 0.06 \times 1{,}000 \qquad \Rightarrow \qquad Coup < YTW \times R_{t_k}$$

Since the bond is a discount bond, its price-to-worst is calculated based on the latest possible redemption:

$$P = 50 \times a_{\overline{20|}0.06} + 1{,}000v^{20} = 50 \times 11.4699 + 311.8047 = \mathbf{885.30}$$

We can use the BA II Plus to answer this question:

20 [*N*] 6 [*I*/*Y*] 50 [*PMT*] 1,000 [*FV*] [CPT] [*PV*]

Result is –885.3008. Answer is **885.30**.

Solution 92

C Chapter 18, Forward Rates

The question is asking for the rate that applies from time 4 to time 5.

$$1 + f_{t-1} = \frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1 + f_4 = \frac{1.095^5}{1.09^4}$$

$$f_4 = \mathbf{0.1152}$$

Solution 93

D Chapter 11, Geometric Progression Annuities

The monthly effective interest rate is:

$$\frac{0.06}{12} = 0.005$$

The annual effective interest rate is:

$$1.005^{12} - 1 = 0.06168$$

The present value of the first year's payments is:

$$2{,}000 a_{\overline{12}|0.005} = 2{,}000 \times \frac{1 - 1.005^{-12}}{0.005} = 23{,}237.8641$$

The payments in the 24 subsequent years increase by 2% per year:

$$P = 23{,}237.8641\left[1 + 1.02v + \cdots + (1.02v)^{24}\right] = 23{,}237.8641 \times \frac{1 - \left(\dfrac{1.02}{1.06168}\right)^{25}}{1 - \dfrac{1.02}{1.06168}}$$

$$= 23{,}237.8641 \times 16.1135 = 374{,}443.38$$

The difference between the lump sum and P is:

$$374{,}500 - P = 374{,}500 - 374{,}443.38 = 56.62$$

Choice D is the correct answer.

Solution 94

A Chapter 3, Accumulated Value

The monthly effective interest rate is:

$$\frac{0.072}{12} = 0.006$$

The equation of value that equates the value of the deposits with $6,500 five years from today is:

$$1{,}700 \times 1.006^{60-n} + 3{,}400 \times 1.006^{60-2n} = 6{,}500$$

$$1{,}700 \times 1.006^{-n} + 3{,}400 \times 1.006^{-2n} = 6{,}500 \times 1.006^{-60}$$

$$3{,}400X^2 + 1{,}700X - 4{,}539.7769 = 0 \quad \text{where: } X = 1.006^{-n}$$

We can use the quadratic formula to solve for X:

$$X = \frac{-1{,}700 \pm \sqrt{1{,}700^2 - 4(3{,}400)(-4{,}539.7769)}}{2 \times 3{,}400}$$

$$X = -1.4323 \quad \text{or} \quad X = 0.93226$$

Using the positive value of X, we have the maximum possible value of n:

$$X = 1.006^{-n}$$

$$0.9323 = 1.006^{-n}$$

$$n = 11.7264$$

Since n must be less than or equal to 11.7264, the maximum integral value of n is **11**.

Solution 95

C Chapter 3, Accumulated Value

We can use the ratio of *S* to *T* to find *d*:

$$\frac{S}{T} = \left(\frac{39}{38}\right)^4$$

$$\frac{1,000\left(1-\frac{d}{2}\right)^{-4}}{1,000\left(1-\frac{d}{4}\right)^{-4}} = \left(\frac{39}{38}\right)^4$$

$$\frac{\left(1-\frac{d}{4}\right)}{\left(1-\frac{d}{2}\right)} = \frac{39}{38}$$

$$39 - 19.5d = 38 - 9.5d$$

$$1 = 10d$$

$$d = 0.1$$

The value of *d* convertible semiannually is equivalent to an annual effective interest rate of *i*:

$$\left(1-\frac{d}{2}\right)^{-2} = 1+i$$

$$0.95^{-2} = 1+i$$

$$i = \mathbf{0.1080}$$

Solution 96

C Chapter 7, Level Annuity

The monthly effective interest rate is:

$$\frac{0.042}{12} = 0.0035$$

May 1, of the year (y + 10) is 124 months after January 1, of the year y. During these 124 months, there are 41 quarterly payments of 100. The equation of value at time 0 is:

$$X + \left[\frac{100}{1.0035^3} + \frac{100}{1.0035^6} + \cdots + \frac{100}{1.0035^{123}}\right] = \frac{1.9X}{1.0035^{124}}$$

$$X + \sum_{k=1}^{41} \frac{100}{1.0035^{3k}} = \frac{1.9X}{1.0035^{124}}$$

The answer is **Choice C**.

Solution 97

D Chapter 7, Level Annuity

Let's work in 2-year periods. For the first six years, the 2-year effective interest rate is:

$$1.04^2 - 1 = 0.0816$$

The accumulated value at time six years of three $100 payments made at the beginning of each 2-year period is:

$$100\ddot{s}_{\overline{3}|8.16\%} = 100\left(\frac{1.0816^3 - 1}{0.0816 / 1.0816}\right) = 351.6778$$

For the last four years, the 2-year effective interest rate is:

$$1.05^2 - 1 = 0.1025$$

The accumulated value at time ten years of the time-6 accumulated value and of the last two $100 payments made at the beginning of each of the remaining 2-year periods is:

$$351.6778(1.1025)^2 + 100\ddot{s}_{\overline{2}|10.25\%}$$
$$= 351.6778 \times 1.2155 + 100\left(\frac{1.1025^2 - 1}{0.1025 / 1.1025}\right)$$
$$= 427.4665 + 231.8006 = 659.2671$$

We use the BA II Plus to obtain the annual effective yield:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

5 [*N*] 100 [*PMT*] –659.2671 [*FV*] [CPT] [*I*/*Y*]

(Result is 9.3636)

[÷] 100 [+] 1 [=] [y^x] 0.5 [=] –1 [=]

Answer is **0.0458.**

Solution 98

C Chapter 7, Level Annuity

The monthly effective interest rate is:

$$1.08^{1/12} - 1 = 0.006434$$

The withdrawals of $25,000 are made at times 15, 16, 17, and 18.

The equation of value at time 0 can be used to find X:

$$X\ddot{a}_{\overline{18\times 12}|0.006434} = \frac{25,000}{1.08^{15}}\ddot{a}_{\overline{4}|0.08}$$

$$X \times \frac{1-1.006434^{-216}}{0.06434} \times 1.06434 = 7,881.0426 \times \frac{1-1.08^{-4}}{0.08} \times 1.08$$

$$X \times 117.2787 = 7,881.0426 \times 3.5771$$

$$X = \mathbf{240.3782}$$

We can use the BA II Plus to answer this question:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

1.08 [y^x] 12 [1/x] [=] [–] 1 [=] [×] 100 [=] [I/Y]

18 [×] 12 [=] [N] 1 [PMT] [CPT] [PV]

(Result is –117.2787) [STO] 1

4 [N] 8 [I/Y] 25,000 [PMT] [CPT] [PV]

(Result is –89,427.4247) [÷] 1.08 [y^x] 15 [=]

[÷] [RCL] 1 [=]

Answer is **240.3782**.

Solution 99

B Chapter 6, Level Annuity

The equation of value at time 0 can be used to find X:

$$15,000\ddot{a}_{\overline{11}|0.10} + \frac{1}{1.10^{10}} \times \frac{15,000}{0.08} = X\ddot{a}_{\overline{10}|0.10} + \frac{X}{1.10^{10}}\ddot{a}_{\overline{15}|0.08}$$

$$15,000 \times \frac{1-1.10^{-11}}{\frac{0.10}{1.10}} + 72,289.37 = X\left[\frac{1-1.10^{-10}}{\frac{0.10}{1.10}} + \frac{1}{1.10^{10}} \times \frac{1-1.08^{-15}}{\frac{0.08}{1.08}}\right]$$

$$15,000 \times 7.1446 + 72,289.37 = X\left[6.7590 + 3.5641\right]$$

$$X = \mathbf{17,384.14}$$

We can use the BA II Plus to answer this question:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

10 [N] 10 [I/Y] 1 [PMT] [CPT] [PV]

(Result is –6.7590) [+/-] [STO] 1

15 [N] 8 [I/Y] 1 [PMT] [CPT] [PV] [÷] 1.10 [y^x] 10 [=]

(Result is –3.5641) [+/-] [STO] 2

11 [N] 10 [I/Y] 15,000 [PMT] [CPT] [PV] [+/-]

[+] 15,000 [÷] 0.08 [÷] 1.10 [y^x] 10 [=]

(Result is 179,457.8734) [÷] [(] [RCL] 1 [+] [RCL] 2 [)] [=]

Answer is **17,384.14**.

Solution 100

A Chapter 8, Varying Annuity

The 6-month effective interest rate is:

$$\frac{0.06}{2} = 0.03$$

Since the last coupon payment was 22.50, the next coupon payment is $X + 22.50$. The present value of the bond is 1,050.50. The time 0 equation of value can be used to solve for X:

$$\frac{22.50 + X}{1.03} + \frac{22.50 + 2X}{1.03^2} + \cdots + \frac{22.50 + 14X}{1.03^{14}} + \frac{300}{1.03^{14}} = 1{,}050.50$$

$$22.50 \times a_{\overline{14}|0.03} + X(Ia)_{\overline{14}|0.03} + \frac{300}{1.03^{14}} = 1{,}050.50$$

$$22.50 \times \frac{1 - 1.03^{-14}}{0.03} + X \times \frac{\ddot{a}_{\overline{14}|} - 14(1.03)^{-14}}{0.03} + 198.3353 = 1{,}050.50$$

$$22.50 \times 11.2960 + X \times \frac{11.6350 - 14(1.03)^{-14}}{0.03} + 198.3353 = 1{,}050.50$$

$$254.1616 + X \times 79.3102 + 198.3353 = 1{,}050.50$$

$$X = \mathbf{7.5401}$$

We can use the BA II Plus to answer this question:

14 [*N*] 3 [*I/Y*] 1 [*PMT*] [CPT] [*PV*]

[×] 1.03 [=] [+/-] [–] 14 [÷] 1.03 [y^x] 14 [=] [÷] 0.03 [=] [STO] 1

22.50 [*PMT*] 300 [*FV*] [CPT] [*PV*] + 1,050.50 [=]

[÷] [RCL] 1 [=]

Answer is **7.54001**.

Solution 101

D Chapter 8, Varying Annuity

Let's use the PIn method find the value of the annuity at the end of 9 years. We have:

$P_1 = 1{,}000$ $\quad I = 500$ $\quad n = 30$

The present value is:

$$PV_9 = \left(P_1 + \frac{I}{i}\right)a_{\overline{n}|} - \frac{In}{i}v^n = \left(1{,}000 + \frac{500}{0.05}\right) \times \frac{1 - (1.05)^{-30}}{0.05} - \frac{500 \times 30}{0.05}(1.05)^{-30}$$

$$= 11{,}000 \times 15.3725 - 69{,}413.2346 = 99{,}683.7267$$

To find the present value at time zero, we discount for 9 years:

$$\frac{99{,}683.7267}{1.05^9} = \mathbf{64{,}257.02}$$

Using the BA II Plus, we have:

30 [N] 5 [I/Y] 1,000 [+] 500 [÷] 0.05 [=] [PMT]

500 [×] 30 [÷] 0.05 [=] [+/-] [FV]

[CPT] [PV]

[÷] 1.05 [y^x] 9 [=]

Result is –64,257.02. Answer is **64,257.02**.

Solution 102

C Chapter 11, Geometric Progression Annuity

The equation of value after 30 years can be used to solve for i:

$$5,000\left[(1+i)^{29}+(1+i)^{28}(1.03)+\cdots+1.03^{29}\right]$$

$$=50,000\left[1+1.03v+\cdots+(1.03v)^{29}\right]$$

$$5,000\times\frac{(1+i)^{29}-\dfrac{1.03^{30}}{1+i}}{1-\dfrac{1.03}{1+i}}=50,000\times\frac{1-\left(\dfrac{1.03}{1+i}\right)^{30}}{1-\dfrac{1.03}{1+i}}$$

$$(1+i)^{29}-\frac{1.03^{30}}{1+i}=10\times\left[1-\left(\frac{1.03}{1+i}\right)^{30}\right]$$

The account balance after

$$(1+i)^{29}\left[1-\left(\frac{1.03}{1+i}\right)^{30}\right]=10\times\left[1-\left(\frac{1.03}{1+i}\right)^{30}\right]$$

$$(1+i)^{29}=10$$

$$i=0.08264$$

the final deposit is:

$$5,000\times\frac{(1+i)^{29}-\dfrac{1.03^{30}}{1+i}}{1-\dfrac{1.03}{1+i}}=5,000\times\frac{(1.08264)^{29}-\dfrac{1.03^{30}}{1.08264}}{1-\dfrac{1.03}{1.08264}}=\mathbf{797,836.82}$$

Solution 103

D Chapter 14, Dividend Discount Model

Although this does not refer to the perpetuity as payments from a share of common stock, we can treat the payments as dividends and use the dividend discount model to find the present value of the payments.

The rate of growth of the quarterly payments is:

$$\frac{2,010}{2,000}-1=0.005$$

The present value of the payments can be used to solve for the quarterly effective interest rate:

$$100,000 = 2,000 + \frac{2,010}{\frac{i^{(4)}}{4} - 0.005}$$

$$\frac{i^{(4)}}{4} = 0.02551$$

The annual effective interest rate is:

$$\left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = 1.02551^4 - 1 = \mathbf{0.1060}$$

Solution 104

A Chapter 10, Continuously Payable Annuity

Let's break the perpetuity into two parts.

The first part consists of the payments made in the first 10 years. The present value of the first part is:

$$\bar{a}_{\overline{10}|} = \frac{1 - v^n}{r} = \frac{1 - 1.06^{-10}}{\ln(1.06)} = 7.5787$$

The second part consists of the payments made after 10 years. The formula for the present value of a continuously payable annuity is:

$$PV_a = \int_a^b Pmt_t e^{-\int_a^t r_s ds} dt$$

The present value at time 10 of the payments made after 10 years is:

$$PV_{10} = \int_{10}^{\infty} 1.03^{t-10} e^{-\int_{10}^{t} \ln(1.06) ds} dt = \int_{10}^{\infty} 1.03^{t-10} e^{-\ln(1.06)(t-10)} dt$$

$$= \int_{10}^{\infty} 1.03^{t-10} \times 1.06^{(10-t)} dt = \left(\frac{1.06}{1.03}\right)^{10} \int_{10}^{\infty} \left(\frac{1.03}{1.06}\right)^t dt$$

$$= \left(\frac{1.06}{1.03}\right)^{10} \left(\frac{1.03}{1.06}\right)^t \frac{1}{\ln\left(\frac{1.03}{1.06}\right)}\Bigg|_{10}^{\infty} = 0 - \left(\frac{1.06}{1.03}\right)^{10} \left(\frac{1.03}{1.06}\right)^{10} \frac{1}{\ln\left(\frac{1.03}{1.06}\right)}$$

$$= \frac{-1}{\ln\left(\frac{1.03}{1.06}\right)} = 34.8309$$

The present value at time 0 of the payments made after 10 years is the present value at time 10, discounted for 10 years:

$$PV_0 = \left(\frac{1}{1.06}\right)^{10} PV_{10} = \left(\frac{1}{1.06}\right)^{10} 34.8309 = 19.4494$$

The present value of the perpetuity is equal to the sum of the present values of the two parts of the perpetuity:

$$7.5787 + 19.4494 = \mathbf{27.0282}$$

Solution 105

C Chapter 5, Varying Force of Interest

The present value at time 5 of the 75,000 payment is:

$$PV_5 = AV_{10}e^{-\int_5^{10} r_s ds} = 75,000 \times e^{-\int_5^{10} \frac{1}{s+1} ds} = 75,000 \times e^{-\ln(s+1)\big|_5^{10}}$$
$$= 75,000 \times e^{\ln(6) - \ln(11)} = 75,000 \times \frac{6}{11}$$

The equation of value at time 0 can be used to solve for X:

$$10,000 + \frac{X}{1.06^3} = 75,000 \times \frac{6}{11} \times \frac{1}{1.06^5}$$
$$X = \mathbf{24,498.79}$$

Solution 106

D Chapter 12, Drop Payments

If we accumulate the initial loan balance to time 2, then we can treat the loan as a loan with the first payment occurring one year later:

$$\frac{15,000,000}{(1-0.055)^2} = 16,796,842.19$$

The annual effective interest rate is:

$$\frac{0.055}{1-0.055} = 0.05820$$

We can solve the time-0 equation of value for n:

$$16,796,842.19 = 1,200,000 a_{\overline{n}|}$$
$$16,796,842.19 = 1,200,000 \times \frac{1-1.05820^{-n}}{0.05820}$$
$$0.1853 = 1.05820^{-n}$$
$$n = 29.7960$$

Therefore, there are 29 payments of 1,200,000 and a final drop payment at time 30:

$$16,796,842.19 = 1,200,000 a_{\overline{29}|} + DropPmt \times 0.945^{30}$$
$$16,796,842.19 = 1,200,000 \times \frac{1-1.05820^{-29}}{0.05820} + DropPmt \times 0.945^{30}$$
$$DropPmt = \mathbf{960,723.59}$$

We can use the BA II Plus to answer this question:

15,000,000 [÷] 0.945 [x^2] [=] [+/-] [*PV*]

0.055 [÷] 0.945 [×] 100 [=] [*I/Y*] 1,200,000 [*PMT*]

[CPT] [*N*]

Result is 29.7960.

29 [*N*] [CPT] [*FV*] [÷] 0.945 [=]

Answer is **960,723.59**.

The closest answer choice is Choice D.

It appears that when the answer choices were prepared for this question, there was an unusually large discrepancy introduced by internal rounding.

Solution 107

C Chapter 12, Loans

We make use of the following formula for the interest portion of a payment:

$$Int_t = (1 - v^{n-t+1})Pmt$$

The formula can be used to find the following expressions:

$$Int_{n-1} = (1 - v^2)Pmt$$
$$Int_{n-3} = (1 - v^4)Pmt$$
$$Int_1 = (1 - v^n)Pmt$$

The interest portion of the payment at time $(n - 1)$ is equal to 0.5250 of the interest portion of the payment at time $(n - 3)$, which allows us to solve for v:

$$(1 - v^2)Pmt = 0.5250(1 - v^4)Pmt$$
$$1 = 0.5250(1 + v^2)$$
$$v = 0.9512$$

The interest portion of the payment at time $(n - 1)$ is equal to 0.1427 of the interest portion of the first payment, which allows us to solve for n:

$$(1 - v^2)Pmt = 0.1427(1 - v^n)Pmt$$
$$0.09524 = 0.1427(1 - v^n)$$
$$0.3326 = v^n$$
$$n = \mathbf{22.00}$$

Solution 108

B Chapter 12, Sinking Funds

The sinking fund payment is:

$$SFP = \frac{L}{s_{\overline{11}|0.047}}$$

The amount owed to the lender is remains constant at *L* until the loan is paid off, so the equation of value at the end of 7 years is:

$$L - \frac{L}{s_{\overline{11}|0.047}} \times s_{\overline{7}|} = 6,241$$

$$L\left[1 - \frac{1.047^7 - 1}{1.047^{11} - 1}\right] = 6,241$$

$$L = 14,749.3182$$

The sinking fund payment is:

$$SFP = \frac{L}{s_{\overline{11}|0.047}} = \frac{14,749.3182}{13.9861} = \mathbf{1,054.57}$$

Solution 109

C Chapter 12, Drop Payments

The level payments satisfy the following time-0 equation of value:

$$200,000 = Pmt \times a_{\overline{360}|0.005}$$

$$200,000 = Pmt \times 166.7916$$

$$Pmt = 1,199.1011$$

When we subtract the present value of the extra payments, the new equation of value is:

$$200,000 - 10,000 \times a_{\overline{5}|1.005^{12}-1} = 1,199.1011 \times a_{\overline{n}|0.005}$$

$$200,000 - 10,000 \times a_{\overline{5}|0.06168} = 1,199.1011 \times \frac{1 - 1.005^{-n}}{0.005}$$

$$200,000 - 10,000 \times 4.1932 = 1,199.1011 \times \frac{1 - 1.005^{-n}}{0.005}$$

$$158,067.9347 = 1,199.1011 \times \frac{1 - 1.005^{-n}}{0.005}$$

$$158,067.9347 = 1,199.1011 \times \frac{1 - 1.005^{-n}}{0.005}$$

$$1.005^{-n} = 0.3409$$

$$n = 215.7768$$

The final payment therefore occurs after 216 months, which is 18 years:

$$\frac{216}{12} = 18$$

The loan originated on January 1, 2003, and the final payment is made 18 years later. Adding 18 years to January 1, 2003 brings us to January 1, 2021. The beginning of January 1, 2021 is the same as the end of **December 31, 2020**.

We can use the BA II Plus to answer this question:

200,000 [*PV*] 360 [*N*] 0.5 [*I*/*Y*] [CPT] [*PMT*]

Result is –1,199.1011.

[STO] 1

5 [*N*] 1.005 [y^x] 12 [–] 1 [=] [×] 100 [=] [*I*/*Y*] 10,000 [*PMT*] [CPT] [*PV*]

[+] 200,000 [=]

Result is 158,067.9347

[*PV*] 0.5 [*I*/*Y*] [RCL] 1 [*PMT*] [CPT] [*N*]

Result is 215.7768, so the final payment occurs after 216 months.

216 [÷] 12 [=]

Result is 18.

[+] 2003 [=]

Result is 2021.

The final payment is made at the beginning of January 1, 2021, which is equivalent to the end of **December 31, 2020**.

Solution 110

D Chapter 12, Loans

The annual effective interest rate is:

$$i = \frac{0.08}{1 - 0.08} = 0.08696$$

We can solve for the amount of the 5 level payments:

$$500{,}000 = Pmt \times a_{\overline{5}|0.08696}$$
$$500{,}000 = Pmt \times 3.9206$$
$$Pmt = 127{,}532.7207$$

If the first 4 payments were instead 128,000, then the balance at the end of 4 years would be:

$$\frac{500{,}000}{0.92^4} - 128{,}000 \times s_{\overline{4}|0.08696} = \frac{500{,}000}{0.92^4} - 128{,}000 \times 4.5526$$
$$= 115{,}202.7473$$

The final payment is the balance at the end of 4 years, accumulated for one additional year of interest:

$$\frac{115,202.7473}{0.92} = \mathbf{125,220.38}$$

We can use the BA II Plus to answer this question:

0.08 [÷] 0.92 [×] 100 [=] [*I*/*Y*]

5 [*N*] 500,000 [+/-] [*PV*] [CPT] [*PMT*]

Result is 127,532.7207.

128,000 [*PMT*] 4 [*N*] [CPT] [*FV*]

[÷] 0.92 [=]

Answer is **125,220.38**.

Solution 111

B Chapter 18, Spot Rates

The price of a zero-coupon bond as a percentage of its redemption value is equal to the inverse of the accumulation factor achieved by investing in the bond.

Therefore investing X in the 6-month bond, for example, results in an accumulated value of:

$$\frac{X}{0.94}$$

Investing X in each of the bonds results in an accumulated value of:

$$\frac{X}{0.94} + \frac{X}{0.95} + \frac{X}{0.96} + \frac{X}{0.97} + \frac{X}{0.98} + \frac{X}{0.99}$$
$$= X\left[\frac{1}{0.94} + \frac{1}{0.95} + \frac{1}{0.96} + \frac{1}{0.97} + \frac{1}{0.98} + \frac{1}{0.99}\right] = 6.2196X$$

Setting this accumulated value equal to 100,000 allows us to solve for X:

$$6.2196X = 100,000$$
$$X = \mathbf{16,078.29}$$

Solution 112

D Chapter 12, Loans

We use the following formulas:

$$Int_t = (1 - v^{n-t+1})Pmt$$
$$Prn_t = v^{n-t+1} \times Pmt$$

Using the information provided in the question, we have:

$$Int_1 = (1 - v^{10})Pmt = 3,600$$
$$Prn_6 = v^5 \times Pmt = 4,871$$

Dividing the first equation by the second equation allows us to solve for v:

$$\frac{(1-v^{10})Pmt}{v^5 \times Pmt} = \frac{3,600}{4,871}$$

$$1 - v^{10} = \frac{3,600}{4,871}v^5$$

$$v^{10} + \frac{3,600}{4,871}v^5 - 1 = 0$$

We can use the quadratic formula to solve for v^5:

$$v^5 = \frac{-\frac{3,600}{4,871} \pm \sqrt{\left(\frac{3,600}{4,871}\right)^2 - 4 \times 1 \times (-1)}}{2}$$

$$v^5 = 0.6966 \quad \text{or} \quad v^5 = -1.4356$$

We use the positive value of v^5 to solve for i:

$$v^5 = 0.6966$$
$$i = 0.07500$$

Since the interest paid in the first year is 3,600, we have:

$$Xi = 3,600$$
$$X \times 0.07500 = 3,600$$
$$X = \mathbf{48,000.18}$$

Solution 113

A Chapter 15, Bonds

The equation of value can be solved for R:

$$P = Coup \times a_{\overline{n|}y} + Rv^n$$

$$10,000 = R \times 0.035 \times a_{\overline{50|}1.0705^{0.5}-1} + \frac{R}{1.0705^{25}}$$

$$10,000 = R\left[0.035 \times \frac{1-1.0705^{-25}}{1.0705^{0.5}-1} + \frac{1}{1.0705^{25}}\right]$$

$$10,000 = R\left[0.035 \times 23.6045 + \frac{1}{1.0705^{25}}\right]$$

$$R = \mathbf{9,917.99}$$

We can use the BA II Plus to answer this question:

1.0705 [y^x] 0.5 [–] 1 [=] [×] 100 [=] [*I*/*Y*]

50 [*N*] 0.035 [*PMT*] 1 [*FV*] [CPT] [*PV*]

Result is –1.008269.

[1/*x*] [×] 10,000 [=]

Result is –9,917.9912. Answer is **9,917.99**.

Solution 114

B Chapter 15, Bonds

At the end of each month, the net cash flow to Jeff is the coupon payment from the bond minus the interest on the loan:

$$10{,}000 \times \frac{0.09}{12} - 2{,}000 \times \frac{0.08}{12} = 61.6667$$

At the end of 10 years, Jeff receives 10,000 from the bond and pays back the 2,000 loan, giving him a net cash flow of:

$$10{,}000 - 2{,}000 = 8{,}000$$

Since the cost of entering this position is 8,000, the time-0 equation of value is:

$$8{,}000 = 61.6667 a_{\overline{120}|\frac{i^{(12)}}{12}} + \frac{8{,}000}{\left(1 + \frac{i^{(12)}}{12}\right)^{120}}$$

We can use the BA II Plus to answer this question:

120 [*N*] 8,000 [+/-] [*PV*]

10,000 [×] 0.09 [÷] 12 [–] 2,000 [×] 0.08 [÷] 12 [=] [*PMT*]

8,000 [*FV*] [CPT] [*I/Y*]

Result is 0.7708.

[÷] 100 [+] 1 [=] [y^x] 12 [–] 1 [=]

Answer is **0.0965**.

Solution 115

B Chapter 15, Bonds

The first equation of value below sets the value of the first bond equal to the value of the second bond, and the second equation sets the value of the second bond equal to the value of the third bond:

$$0.0528 \times 1{,}000 \times a_{\overline{n}|} + 1{,}000v^n = 0.0440 \times 1{,}100 \times a_{\overline{n}|} + 1{,}100v^n$$

$$0.0440 \times 1{,}100 \times a_{\overline{n}|} + 1{,}100v^n = 1{,}320r \times a_{\overline{n}|} + 1{,}320v^n$$

The two equations above can be simplified as follows:

$$4.4 \times a_{\overline{n}|} = 100v^n$$

$$(48.4 - 1{,}320r) \times a_{\overline{n}|} = 220v^n$$

Dividing the first equation into the second equation allows us to solve for r:

$$\frac{(48.4-1{,}320r)\times a_{\overline{n}|}}{4.4\times a_{\overline{n}|}}=\frac{220v^n}{100v^n}$$

$$\frac{48.4-1{,}320r}{4.4}=\frac{220}{100}$$

$$48.4-1{,}320r=9.68$$

$$r=\mathbf{0.02933}$$

Solution 116

A Chapter 15, Bonds

The time-0 equation of value can be used to solve for c:

$$582.53=\frac{250}{1.04^6}+c\left[\frac{1.02}{\sqrt{1.04}}+\left(\frac{1.02}{\sqrt{1.04}}\right)^2+\cdots+\left(\frac{1.02}{\sqrt{1.04}}\right)^{12}\right]$$

$$384.9514=c\times\frac{\frac{1.02}{\sqrt{1.04}}-\left(\frac{1.02}{\sqrt{1.04}}\right)^{13}}{1-\frac{1.02}{\sqrt{1.04}}}$$

$$c=\mathbf{32.04}$$

Solution 117

E Chapter 15, Bonds

We can use the book values provided to find the coupon amount:

$$BV_{t+k}=BV_t(1+y)^k-Coup\times s_{\overline{k}|y}$$

$$BV_4=BV_3(1.06)-Coup\times s_{\overline{1}|0.06}$$

$$1{,}277.38=1{,}254.87(1.06)-Coup\times 1$$

$$Coup=52.7822$$

We can now use the prospective formula to find n:

$$BV_t = Coup \times a_{\overline{n-t}|y} + Rv^{n-t}$$

$$BV_3 = 52.7822 \times a_{\overline{n-3}|0.06} + 1,890v^{n-3}$$

$$1,254.87 = 52.7822 \times \frac{1 - 1.06^{-(n-3)}}{0.06} + 1,890v^{n-3}$$

$$1,254.87 = 879.7033\left(1 - v^{n-3}\right) + 1,890v^{n-3}$$

$$375.1667 = 1,010.2967v^{n-3}$$

$$0.3713 = 1.06^{3-n}$$

$$3 - n = -17.0010$$

$$n = \mathbf{20.00}$$

We can use the BA II Plus to answer this question:

1,254.87 [×] 1.06 [–] 1,277.38 [=]

Result is 52.7822.

[*PMT*] 6 [*I*/*Y*] 1,254.87 [+/-] [*PV*] 1,890 [*FV*]

[CPT] [*N*]

Result is 17.0010.

[+] 3 [=]

Answer is **20.00**.

Solution 118

A Chapter 15, Bonds

The discount accrued with the fourth coupon is 8.44:

$$BV_t = BV_{t-1} + DA_t$$

$$BV_4 = BV_3 + DA_4$$

$$DA_4 = BV_4 - BV_3$$

$$DA_4 = 8.44$$

The discount accrued can also be written as:

$$DA_t = (Ry - Coup)v^{n-t+1}$$

We can now solve for n:

$$8.44 = (2,500 \times 0.04 - 2,500 \times 0.035)v^{n-4+1}$$

$$0.6752 = 1.04^{3-n}$$

$$3 - n = -10.0137$$

$$n = 13.0137$$

The n that was found above is the number of coupons paid, but the question asks for the number of years until the bond matures. Therefore, we must divide the n above by 2:

$$\frac{13.0137}{2} = \mathbf{6.51}$$

Solution 119

C Chapter 18, Forward Rates

The 3-year accumulation factor calculated with the spot rate is the same as the 3-year accumulation factor calculated with the forward rates:

$$(1+s_t)^t = (1+f_0)(1+f_1)\cdots(1+f_{t-1})$$
$$(1+s_3)^3 = (1+f_0)(1+f_1)(1+f_2)$$
$$(1+s_3)^3 = 1.04 \times 1.06 \times 1.08$$
$$s_3 = \mathbf{0.0599}$$

Solution 120

D Chapter 13, Time-Weighted & Dollar-Weighted ROR

The income and fund exposure are:

$$\begin{aligned}\text{Income} &= \text{Withdrawals} - \text{Deposits} = 55{,}000 + 10{,}000 - 50{,}000 - 8{,}000 \\ &= 7{,}000 \\ \text{Fund exposure} &= \sum(\text{Net deposit})(\text{Time deposit is in the fund}) \\ &= 50{,}000 + 8{,}000 \times \frac{8}{12} - 10{,}000(1-t) \\ &= 45{,}333.3333 + 10{,}000t\end{aligned}$$

The dollar-weighted rate of return is equal to the time-weighted rate of return:

$$\frac{7{,}000}{45{,}333.3333 + 10{,}000t} = \frac{52{,}000}{50{,}000} \times \frac{62{,}000}{60{,}000} \times \frac{55{,}000}{52{,}000} - 1$$
$$\frac{7{,}000}{45{,}333.3333 + 10{,}000t} = 0.1367$$
$$45{,}333.3333 + 10{,}000t = 51{,}219.5122$$
$$t = \mathbf{0.5886}$$

Solution 121

B Chapter 16, Duration

The formula for Macaulay Duration is:

$$MacDur = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)}$$

We can use the duration of the 3-year annuity to solve for v:

$$0.93 = \frac{0 + v + 2v^2}{1 + v + v^2}$$

$$0.93 + 0.93v + 0.93v^2 = v + 2v^2$$

$$1.07v^2 + 0.07v - 0.93 = 0$$

$$v = \frac{-0.07 \pm \sqrt{0.07^2 - 4(1.07)(-0.93)}}{2 \times 1.07}$$

$$v = 0.9002 \quad \text{or} \quad v = -0.9656$$

We use the positive discount factor to find the duration of Annuity B:

$$\frac{0 + v + 2v^2 + 3v^3}{1 + v + v^2 + v^3} = \frac{0.9002 + 2 \times 0.9002^2 + 3 \times 0.9002^3}{1 + 0.9002 + 0.9002^2 + 0.9002^3} = \frac{4.7088}{3.4398} = \mathbf{1.3689}$$

Solution 122

D Chapter 16, Duration

The Macaulay duration is:

$$MacDur = \frac{\sum_{t>0} \left[t \times PV_0\left(CF_t\right) \right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{40 \times 2}{1.07^2} + \frac{25 \times 3}{1.07^3} + \frac{100 \times 4}{1.07^4}}{\frac{40}{1.07^2} + \frac{25}{1.07^3} + \frac{100}{1.07^4}} = \frac{436.2555}{131.6345} = \mathbf{3.314}$$

Solution 123

B Chapter 16, Duration

As of 10/6/2015, the Exam FM Sample Solutions available on the Society of Actuaries' web site incorrectly showed the answer as being Choice C.

The Macaulay duration of Bond A is:

$$MacDur_A = \ddot{a}_{\overline{3|}} = \frac{1 - 1.10^{-3}}{0.10} \times 1.10 = 2.7355$$

The modified duration of Bond A is:

$$ModDur_A = \frac{MacDur_A}{1 + \frac{y^{(m)}}{m}} = \frac{2.7355}{1.10} = 2.4869$$

The modified duration of Bond B is equal to the modified duration of Bond A:

$$ModDur_B = 2.4869$$

The Macaulay duration of Bond B is found below:

$$ModDur_B = \frac{MacDur_B}{1+\frac{y^{(m)}}{m}}$$

$$2.4869 = \frac{MacDur_B}{1+\frac{0.10}{2}}$$

$$MacDur_B = 2.4869 \times 1.05$$

$$MacDur_B = \mathbf{2.6112}$$

Solution 124

C Chapter 16, Duration

The price and the derivative of the price can be used to obtain an expression for the modified duration:

$$P = 1 + \frac{1}{y}$$

$$P' = -y^{-2}$$

$$ModDur = -\frac{P'}{P} = -\frac{-y^{-2}}{1+y^{-1}} = \frac{1}{y^2+y} = \frac{1}{y(1+y)}$$

The relationship between the modified duration and the Macaulay duration can be used to find the yield:

$$ModDur = \frac{MacDur}{1+y}$$

$$\frac{1}{y(1+y)} = \frac{30}{(1+y)}$$

$$\frac{1}{y} = 30$$

$$y = \frac{1}{30}$$

The modified duration is:

$$ModDur = \frac{1}{y(1+y)} = \frac{1}{\frac{1}{30}\left(1+\frac{1}{30}\right)} = \mathbf{29.03}$$

Solution 125

D Chapter 14, Common Stock

Let Div_J be the next dividend of Stock J. The value of Stock F is twice the value of Stock J:

$$\text{Value of Stock F} = 2 \times (\text{Value of Stock J})$$
$$\frac{0.5Div_J}{0.088 - g} = 2 \times \frac{Div_J}{0.088 + g}$$
$$2(0.088 - g) = 0.5(0.088 + g)$$
$$4(0.088 - g) = 0.088 + g$$
$$3 \times 0.088 = 5g$$
$$g = \mathbf{0.0528}$$

Solution 126

B Chapter 17, Immunization

Statement I is false because an inverted yield curve does not reduce the need for immunization.

Statement II is true, because the modified duration is equal to the Macaulay duration divided by a one-period accumulation factor. Therefore, if either the modified or Macaulay durations are equal, then the other will be equal as well.

Statement III is false, because exact matching is accomplished only when the cash flows themselves are matched.

Solution 127

A Chapter 17, Immunization

The present value, Macaulay duration, and Macaulay convexity of the liabilities are:

$$PV_L = \frac{500}{1.10} + \frac{1{,}000}{1.10^4} = 1{,}137.5589$$

$$MacDur_L = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{500}{1.10} \times 1 + \frac{1{,}000}{1.10^4} \times 4}{1{,}137.5589} = 2.8013$$

$$MacC_L = \frac{\sum_{t>0}\left[t^2 \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{500}{1.10} \times 1^2 + \frac{1{,}000}{1.10^4} \times 4^2}{1{,}137.5589} = 10.0063$$

Since the duration of the assets is equal to the duration of the liabilities, the average duration of the assets is 2.8013. Let the weight of the cash flow of X be w:

$$0 \times w + (1 - w) \times 3 = 2.8013$$
$$w = 0.06625$$

Since the cash flow of X occurs at time 0, the value of X is its weight times the present value of the liabilities:

$$X = 0.06625 \times 1{,}137.5589 = \mathbf{75.36}$$

The Macaulay convexity of the assets is:

$$MacC_A = \frac{\sum_{t>0}\left[t^2 \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = w \times 0^2 + (1-w) \times 3^2$$
$$= 0.06625 \times 0 + (1 - 0.06625) \times 9 = 8.4038$$

Since the Macaulay convexity of the assets is less than the Macaulay convexity of the liabilities, the conditions of Redington immunization are not satisfied. **Choice A** is the correct answer.

Solution 128

D Chapter 17, Full Immunization

Let w be the weight of the first asset cash flow:

$$w = \frac{\frac{300{,}000}{1.04^6}}{\frac{1{,}000{,}000}{1.04^8}} = 0.32448$$

The Macaulay duration of the assets must be equal to the Macaulay duration of the liability:

$$6w + y(1-w) = 8$$
$$6(0.32448) + y(1 - 0.32448) = 8$$
$$y = 8.9607$$

The present value of the assets must be equal to the present value of the liabilities:

$$\frac{300{,}000}{1.04^6} + \frac{X}{1.04^{8.9607}} = \frac{1{,}000{,}000}{1.04^8}$$
$$X = \mathbf{701{,}458.26}$$

Solution 129

A Chapter 17, Immunization

The present value, Macaulay duration, and Macaulay convexity of the liabilities are:

$$PV_L = \frac{573}{1.07^2} + \frac{701}{1.07^5} = 1{,}000.2837$$

$$MacDur_L = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{573}{1.07^2}\times 2 + \frac{701}{1.07^5}\times 5}{1{,}000.2837} = 3.4990$$

$$MacC_L = \frac{\sum_{t>0}\left[t^2 \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{573}{1.07^2}\times 2^2 + \frac{701}{1.07^5}\times 5^2}{1{,}000.2837} = 14.4929$$

The present value, Macaulay duration, and Macaulay convexity of Choice A are:

$$PV_A = 500 + 500 = 1{,}000$$

$$MacDur_A = 0.5 \times 1 + 0.5 \times 6 = 3.5$$

$$MacC_A = 0.5 \times 1^2 + 0.5 \times 6^2 = 18.5$$

Since the present value and the Macaulay duration of Choice A are equal (within rounding tolerance) to the present value and Macaulay duration of the liabilities, and the Macaulay convexity of Choice A exceeds the Macaulay convexity of the liabilities, **Choice A is the correct answer**.

Choice B's Macaulay duration can be found as the weighted average of the timing of its cash flows:

$$MacDur0_B = 0.572 \times 1 + 0.428 \times 6 = 3.14$$

Since 3.14 is not equal to the duration of the liabilities, Choice B is not correct.

Choices C and D have present values in excess of the present value of the liabilities, so neither is correct.

Choice E has a single cash flow, so its Macaulay convexity is equal to the square of the time of the cash flow:

$$MacC_E = 3.5^2 = 12.25$$

Since 12.25 is less than the Macaulay convexity of the liabilities, Choice E is not correct.

Solution 130

D Chapter 17, Immunization

The Macaulay duration of the liabilities is:

$$MacDur_L = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{\frac{402.11}{1.10}\times 1+\frac{402.11}{1.10^2}\times 2+\frac{402.11}{1.10^3}\times 3}{1,000} = 1.9365$$

The Macaulay duration of the assets must be equal to the Macaulay duration of the liabilities. Let w be weight of the one-year bond:

$$w + 3(1-w) = 1.9365$$
$$w = 0.53173$$

The amounts to invest in the one-year bond and the 3-year bond are:

One-year bond: $1,000w = 1,000 \times 0.53173 =$ **531.73**

Two-year bond: $1,000(1-w) = 1,000(1-0.53173) =$ **468.26**

Solution 131

A Chapter 17, Immunization

All of the answer choices have the same present value as the present value of the liabilities, so we'll focus on the requirements that the Macaulay duration of the assets be equal to that of the liabilities and that the Macaulay convexity of the assets be greater than that of the liabilities.

For Choice A, we have:

$$MacD_A = \frac{3,077}{9,697}\times 5+\frac{6,620}{9,697}\times 20 = 15.2403$$
$$MacC_A = \frac{3,077}{9,697}\times 5^2+\frac{6,620}{9,697}\times 20^2 = 281.0070$$

Since the Macaulay duration of the assets is equal (within rounding tolerance) to the Macaulay duration of the liabilities, and the Macaulay convexity of the assets is greater than the Macaulay convexity of the liabilities, **Choice A** is the correct answer.

For Choice B, the Macaulay duration is not equal to the Macaulay duration of the liabilities:

$$MacD_B = \frac{6,620}{9,697}\times 5+\frac{3,077}{9,697}\times 20 = 9.7597$$

For Choice C, the Macaulay duration is not equal to the Macaulay duration of the liabilities:

$$MacD_C = \frac{465}{9,697}\times 15+\frac{9,232}{9,697}\times 20 = 19.7602$$

For Choice D, the Macaulay duration is not equal to the Macaulay duration of the liabilities:

$$MacD_D = \frac{4,156}{9,697} \times 15 + \frac{5,541}{9,697} \times 20 = 17.8571$$

For Choice E, the Macaulay convexity is not greater than the Macaulay convexity of the liabilities:

$$MacD_E = \frac{9,232}{9,697} \times 15 + \frac{465}{9,697} \times 20 = 15.2398$$

$$MacC_E = \frac{9,232}{9,697} \times 15^2 + \frac{465}{9,697} \times 20^2 = 233.3918$$

Solution 132

E Chapter 17, Asset-Liability Management

The liability cash flows occur at time 1 and time 2 in the following amounts:

Time 1: $20,000 \times 1.10 \times 0.5 = 11,000$

Time 2: $(20,000 \times 1.10 \times 0.5) \times 1.10 = 12,100$

The strategy that costs the least to implement produces the highest profit. To answer this question, we:

1. Determine the cost of each strategy.
2. Find the lowest cost strategy that provides the necessary cash flows.

The cost of each strategy is listed below:

A: $9,091 + 8,264 + 2,145 = 19,500$

B: $10,000 + 10,000 = 20,000$

C: $10,000 + 9,821 = 19,821$

D: $8,910 + 731 + 10,000 = 19,641$

E: $8,821 + 10,804 = 19,625$

Choice A has the lowest cost, so we check to see if it provides the necessary cash flows:

Time 1: $9,091 \times 1.10 + 2,145 \times 0.12 = 10,257.50$

Time 2: $8,264 \times 1.11^2 + 2,145 \times 1.12 = 12,584.4744$

Since Choice A does not provide at least 11,000 in year 1, it is not correct.

Since Choice E has the next lowest cost, we check to see if provides the necessary cash flows:

Time 1: $8,821 \times 1.10 + 10,804 \times 0.12 = 10,999.58$

Time 2: $10,804 \times 1.12 = 12,100.48$

Choice E provides (within rounding tolerance) the necessary cash flows to make the liability payments, so **Choice E** is the correct answer.

Solution 133

C Chapter 17, Dedication

We do not use the 4.50% bond, because:

1. It has the lowest yield, and
2. It produces cash flow at time 1, reducing our ability the use the highest-yielding bond, which is the 6.75% bond.

The smallest cost of assets that provide the necessary cash flows is:

$$\frac{20{,}000}{1.05^2} + \frac{25{,}000}{1.0675} = \mathbf{41{,}559.79}$$