November 2005 Exam Solutions

Solution 1

D Chapter 13, Project Evaluation

The cash flows that occur at the middle of the year are the sales revenue, salaries paid, and other expenses paid. Net investment income is assumed to be received at the end of the year.

The net investment income is based on the simple interest rate of 8% and the time that the funds are available to earn interest:

$$\text{Net investment income} = 0.08 \times (\text{Beg. of Year Assets}) + 0.08 \times 0.50 \times (\text{Mid-year cash flow})$$

$$2,000,000 = 0.08 \times 25,000,000 + 0.08 \times 0.5 \times (X - 2,200,000 - 750,000)$$

$$X = 2,950,000$$

The mid-year cash flows net to 0:

$$\text{Sales revenue} - \text{Salaries paid} - \text{Other expenses paid} = 2,950,000 - 2,200,000 - 750,000 = 0$$

The assets at the end of the year are equal to the assets at the beginning of the year plus the net mid-year cash flow plus the net investment income:

$$25,000,000 + 0 + 2,000,000 = 27,000,000$$

The annual effective yield is relatively easy to find, because the mid-year cash flows net to 0:

$$25,000,000(1+i) + 0(1+i)^{0.5} = 27,000,000$$

$$i = \frac{2,000,000}{25,000,000}$$

$$i = \mathbf{0.08}$$

Solution 2

C Chapter 16, Duration

The Macaulay duration is:

$$MacDur = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{10v + 20v^2 + \cdots + 70v^7 + 80v^8 + 800v^8}{10v + 10v^2 + \cdots + 10v^8 + 100v^8}$$

$$= \frac{10(Ia)_{\overline{8}|0.08} + \dfrac{800}{1.08^8}}{10a_{\overline{8}|0.08} + \dfrac{100}{1.08^8}} = \frac{10 \times \dfrac{6.2064 - 8v^8}{0.08} + 432.2151}{10 \times 5.7466 + 54.0269}$$

$$= \frac{10 \times 23.5527 + 432.2151}{57.4664 + 54.0269} = \mathbf{5.9891}$$

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Alternatively, we can use the BA II Plus cash flow worksheet to answer this question:

8 [*N*] 8 [*I/Y*] 10 [*PMT*] 100 [*FV*]

[CPT] [*PV*]

(Result is -111.4933)

[CF] ↓

10 [ENTER] ↓↓

20 [ENTER] ↓↓

30 [ENTER] ↓↓

40 [ENTER] ↓↓

50 [ENTER] ↓↓

60 [ENTER] ↓↓

70 [ENTER] ↓↓

880 [ENTER]

[NPV] 8 [ENTER] ↓ [CPT]

(Result is 667.7425)

[÷] [RCL] [*PV*] [=]

(Result is -5.9891)

Answer is **5.9891**

To exit the cash flow worksheet:

[2nd] [QUIT]

Solution 3

C Chapter 7, Level Annuities

The equation of value at the end of 7 years is:

$$\left[50\ddot{s}_{\overline{24}|j}(1+i)^4+100\ddot{s}_{\overline{24}|j}(1+i)^2+150\ddot{s}_{\overline{24}|j}\right](1+i)=10{,}000$$

The equation can be rearranged to match Choice **C**:

$$\left[\ddot{s}_{\overline{24}|j}(1+i)^4+2\ddot{s}_{\overline{24}|j}(1+i)^2+3\ddot{s}_{\overline{24}|j}\right](1+i)=200$$

$$\ddot{s}_{\overline{24}|j}(1+i)\left[(1+i)^4+2(1+i)^2+3\right]=200$$

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Solution 4

D Chapter 15, Bonds

The formula for the price of the bond can be used to find the redemption value:

$$P = Coup \times a_{\overline{n}|y} + Rv^n$$

$$118.20 = 4 \times a_{\overline{20}|0.03} + \frac{R}{1.03^{20}}$$

$$118.20 = 4 \times 14.8775 + \frac{R}{1.03^{20}}$$

$$R = \mathbf{106.00}$$

We can use the BA II Plus to answer this question:

20 [*N*] 3 [*I*/*Y*] 118.20 [+/-] [*PV*] 4 [*PMT*]

[CPT] [*FV*]

Answer is **106.00**.

Solution 5

Call Options

This question was not scored, because it is based on material that was not on the syllabus when the exam was administered.

Solution 6

A Chapter 18, Forward Rates

The forward rate j applies from time 4 to time 5:

$$j = f_{4,1} = f_4 = \frac{(1+s_5)^5}{(1+s_4)^4} - 1 = \frac{(1+0.09+0.002\times 5-0.001\times 25)^5}{(1+0.09+0.002\times 4-0.001\times 16)^4} - 1$$

$$= \frac{1.075^5}{1.082^4} - 1 = \mathbf{0.04745}$$

Solution 7

D Chapter 18, Spot Rates

Lending annually at 4.00% for 6 years is clearly an inferior strategy, because 4.00% is the lowest possible rate. Likewise lending for 3 years at 4% and for 3 years at 5.00% is clearly inferior to lending for 6 years at 5.00%.

The remaining viable strategies that must be compared are:

- Lend for 6 years at 5.00%. This results in an accumulation factor of:

$$\left(1 + \frac{0.05}{4}\right)^{24} = 1.3474$$

- Lend for 1 year at 4.00% and 5 years at 5.65%. This results in an accumulation factor of:

$$\left(1+\frac{0.04}{4}\right)^4\left(1+\frac{0.0565}{4}\right)^{5\times 4}=1.3776$$

Since the second strategy has a higher accumulation factor, it produces the maximum annual effective rate, which is:

$$1.3776^{1/6}-1=\mathbf{0.0548}$$

Solution 8

B Chapter 11, Geometric Varying Annuities

The 20 payments are described below:

Time	Payment
0	100
1	100×1.05
2	100×1.05^2
…	…
9	100×1.05^9
10	$100\times 1.05^9\times 0.95$
11	$100\times 1.05^9\times 0.95^2$
…	…
19	$100\times 1.05^9\times 0.95^{10}$

Let's begin by finding the present value of the first 10 payments:

$$100+100(1.05)v+\cdots+100(1.05)^9v^9=100\times\frac{1-\left(\frac{1.05}{1.07}\right)^{10}}{1-\frac{1.05}{1.07}}=919.9462$$

The present value of the second set of 10 payments is:

$$100(1.05)^9v^{10}(0.95)+100(1.05)^9v^{11}(0.95)^2+\cdots+100(1.05)^9v^{19}(0.95)^{10}$$

$$=100(1.05)^9v^9\times\frac{\frac{0.95}{1.07}-\left(\frac{0.95}{1.07}\right)^{11}}{1-\frac{0.95}{1.07}}=464.6992$$

The present value of the payments is the sum of the present value of the first 10 payments and the second 10 payments:

$$919.9462+464.6992=\mathbf{1{,}384.65}$$

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Solution 9

E Chapter 14, Dividend Discount Model

The equation of value at time 0 can be used to solve for i. To find the present value of the payments received by the company, we can use the dividend discount model:

$$1,000+\frac{150}{i}=\frac{100}{i-0.05}$$

$$1,000i(i-0.05)+150(i-0.05)=100i$$

$$1,000i^2-50i+150i-7.5-100i=0$$

$$1,000i^2=7.5$$

$$i=\sqrt{\frac{7.5}{1,000}}$$

$$i=\mathbf{0.0866}$$

Solution 10

C Chapter 17, Dedication

The yields on the zero-coupon bonds are the spot yields, and we can use the spot yields to find the present value of the liabilities:

$$\frac{1,000}{1.10}+\frac{2,000}{1.12^2}=\mathbf{2,503.48}$$

Solution 11

B Chapter 15, Bonds

The bond price is:

$$P=Coup\times a_{\overline{n}|y}+Rv^n=40a_{\overline{20}|0.03}+\frac{1,000}{1.03^{20}}=40\times\frac{1-1.03^{-20}}{0.03}+553.6748$$
$$=40\times14.8775+553.6748=1,148.7747$$

Since the investor borrows the purchase price of the bond at an annual effective rate of 5%, the amount to be repaid at the end of 10 years is:

$$1,148.7747\times1.05^{10}=1,871.2330$$

The proceeds from the invested coupons and the redemption value of the bond at time 10 years is:

$$Coup\times s_{\overline{n}|}+R=40s_{\overline{20}|0.02}+1,000=40\times\frac{1.02^{20}-1}{0.02}+1,000$$
$$=40\times24.2974+1,000=1,971.8948$$

Her net cash flow at the end of 10 years is equal to her net gain:

$$1,971.8948-1,871.2330=\mathbf{100.66}$$

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We can use the BA II Plus to answer this question:

20 [*N*] 3 [*I*/*Y*] 40 [*PMT*] 1,000 [*FV*]

[CPT] [*PV*] [$\times$] 1.05 [y^x] 10 [=] [STO] 1

(Result is –1,871.2330)

2 [*I*/*Y*] 0 [*PV*] [CPT] [*FV*] [–] 1,000 [=] [+/-]

(Result is 1,971.8948)

[+] [RCL] 1 [=]

Answer is **100.66**.

Solution 12

B Chapter 8, Varying Annuities

The value of Megan's perpetuity can be used to find the interest rate:

$$3,250 = \frac{130}{i}$$
$$i = 0.04$$

Using the PIn method, we have the following values for Chris's annuity:

$$P_1 = P \qquad I = 15 \qquad n = 20$$

The formula for the present value can be used to find the value of the first payment:

$$PV_0 = \left(P_1 + \frac{I}{i}\right)a_{\overline{n|}} - \frac{In}{i}v^n$$
$$3,250 = \left(P + \frac{15}{0.04}\right) \times \frac{1-(1.04)^{-20}}{0.04} - \frac{15 \times 20}{0.04}(1.04)^{-20}$$
$$3,250 = (P + 375) \times 13.5903 - 3,422.9021$$
$$P = \mathbf{116.0038}$$

Using the BA II Plus, we have:

20 [*N*] 4 [*I*/*Y*] 3,250 [+/-] [*PV*] 15 [$\times$] 20 [$\div$] 0.04 [=] [+/-] [*FV*]

[CPT] [*PMT*]

(Result is 491.0038)

[–] 15 [$\div$] 0.04 [=]

Answer is **116.0038**.

Solution 13

A Chapter 7, Level Annuities

We can use the BA II Plus to answer this question:

40 [*N*] 10,000 [+/-] [*PV*] 400 [*PMT*] [CPT] [*I*/*Y*]

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(Result is 2.5244)

[÷] 100 [+] 1 [=] [y^x] 3 [1/x] [=] [–] 1 [=] [×] 12 [=]

Answer is **0.1001**.

Solution 14

B Chapter 8, Varying Annuities

There are two funds, one earning 8% and one earning 6%. The deposits into the funds are described below:

Time	8%	6%
0	X	
1	X	$X \times 0.08$
2	X	$2X \times 0.08$
…	…	…
19	X	$19X \times 0.08$
20		$20X \times 0.08$

Since the interest from the 8% fund is paid into the 6% fund, the value of the 8% fund at the end of 20 years is 20X. The accumulated value of the 6% fund can be found using an increasing annuity immediate:

$$20X + (Is)_{\overline{20}|0.06} \times 0.08X = 5,600$$

$$20X + \frac{\ddot{s}_{\overline{20}|0.06} - 20}{0.06} \times 0.08X = 5,600$$

$$20X + \frac{\dfrac{1.06^{20} - 1}{0.06 / 1.06} - 20}{0.06} \times 0.08X = 5,600$$

$$20X + \frac{38.9927 - 20}{0.06} \times 0.08X = 5,600$$

$$20X + 316.5454 \times 0.08X = 5,600$$

$$X = \mathbf{123.56}$$

We can use the BA II Plus to obtain the value of $(Is)_{\overline{20}|0.06}$:

20 [N] 6 [I/Y] 1 [PMT] [CPT] [FV] [×] 1.06 [+/-]

[–] 20 [=] [÷] 0.06

Result is 316.5454.

Solution 15

B Chapter 18, Forward Rates

Although this question tells us to use forward rates, the spot rates produce the same values as the forward rates, so there is no need to separately calculate the forward rates.

The present value at time 0 of the annuity is:

$$PV_0 = \frac{5,000}{1.0575^2} + \frac{5,000}{1.0625^3} + \frac{5,000}{1.0650^4} = 12,526.1951$$

The present value at time 0 can be accumulated for one year using the one-year spot rate to find the current value of the annuity in one year:

$$CV_1 = 12,526.1951 \times 1.05 = \mathbf{13,152.50}$$

Solution 16

C Chapter 15, Bonds

At the end of 10 years, the accumulated value of the proceeds from the bond is:

$$AV_{10} = 45s_{\overline{20}|0.035} + 1,000 = 45 \times 28.2797 + 1,000 = 2,272.5857$$

The 6-month effective yield, y, equates the present value of the proceeds with the purchase price:

$$925 = \frac{2,272.5857}{(1+y)^{20}}$$

$$y = 0.04597$$

The nominal yield convertible semiannually is twice the 6-month effective yield:

$$2y = 2 \times 0.04597 = \mathbf{0.0919}$$

We can use the BA II Plus to answer this question:

20 [*N*] 3.5 [*I*/*Y*] 45 [*PMT*] [CPT] [*FV*]

Result is –1,272.5857.

[–] 1,000 [=] [*FV*] 0 [*PMT*] 925 [*PV*] [CPT] [*I*/*Y*]

Result is 4.5969

[×] 2 [=]

Answer is **9.19%**.

Solution 17

D Short Sales

The proceeds from selling the stock short are not received until the position is closed out by purchasing it at the end of the year. The yield on the transaction can be used to solve for X:

$$0.25 = \frac{25,000 - X + 0.08 \times 25,000 \times 0.40}{25,000 \times 0.40}$$

$$X = \mathbf{23,300}$$

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Solution 18

D Chapter 12, Loans

The amount of each level payment is:

$$Pmt = 789 + 211 = 1{,}000$$

The principal repaid in the 18th payment is found below:

$$\frac{Prn_{t+k}}{Prn_t} = (1+i)^k$$

$$\frac{Prn_{18}}{211} = (1.07)^{10}$$

$$Prn_{18} = 415.0689$$

The interest paid in the 18th payment is the payment minus the principal paid:

$$1{,}000 - 415.0689 = \mathbf{584.93}$$

Solution 19

E Chapter 18, Spot Rates

I. True. Zero-coupon bonds can be created by stripping off and selling the individual coupon payments of bond.

II. True. The yields on stripped treasuries are spot rates, and they are also the yield rates of the zero-coupon bonds.

III. False. The interest rates on the risk-free yield curve are not forward rates. If the risk-free yield curve is constructed using zero-coupon bonds, then the interest rates on the risk-free yield curve are spot rates.

Solution 20

D Chapter 14, Dividend Discount Model

At the end of 10 years, the present value of the remaining payments is found using the dividend discount model:

$$PV_{10} = \frac{2(1.02)}{0.06 - 0.02} = 51$$

The present value of the payments at time 0 is:

$$\begin{aligned} PV_0 &= 1a_{\overline{5|}} + 2a_{\overline{5|}} \times v^5 + 51v^{10} \\ &= \left(1 + 2v^5\right) a_{\overline{5|}} + 51v^{10} \\ &= \left(1 + 2(1.06)^{-5}\right)\frac{1 - 1.06^{-5}}{0.06} + 51(1.06)^{-10} \\ &= (1 + 2 \times 0.7473)4.2124 + 51 \times 0.5584 \\ &= \mathbf{38.99} \end{aligned}$$

Solution 21

D Chapter 17, Immunization

I. False. Classical immunization calls for the convexity of the assets to be greater than the convexity of the liabilities.

II. True. Full immunization protects the surplus from both large and small shifts in the yield.

III. True. Immunization is used to protect the surplus from changes in the interest rates.

Solution 22

B Chapter 15, Callable Bonds

We can use the BA II Plus to obtain the number of 6-month periods that the bond was held. We then divide by 2 to obtain the number of years that the bond was held:

5 [*I*/*Y*] 918 [+/-] [*PV*] 45 [*PMT*] 1,100 [*FV*]

[CPT] [*N*]

Result is 49.3531.

[÷] 2 [=]

Answer is **24.68**.

Solution 23

C Chapter 8, Varying Annuities

Using the PIn method, we have:

$P_1 = 2{,}500 \qquad I = -100 \qquad n = 25$

The present value is:

$$PV_0 = \left(P_1 + \frac{I}{i}\right)a_{\overline{n}|} - \frac{In}{i}v^n = \left(2{,}500 - \frac{100}{0.10}\right) \times \frac{1-(1.10)^{-25}}{0.10} + \frac{100 \times 25}{0.10}(1.10)^{-25}$$
$$= 1{,}500 \times 9.0770 + 2{,}307.4000$$
$$= \mathbf{15{,}922.96}$$

Using the BA II Plus, we have:

25 [*N*] 10 [*I*/*Y*] 2,500 [–] 100 [÷] 0.10 [=] [*PMT*]

100 [×] 25 [÷] 0.10 [=] [*FV*]

[CPT] [*PV*]

(Result is –15,922.96)

Answer is **15,922.96**.

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Solution 24

C Chapter 15, Bonds

We can use the BA II Plus to obtain the quarterly effective yield. We then multiply by 4 to obtain the nominal yield convertible quarterly:

120 [*N*] 850 [+/-] [*PV*] 30 [*PMT*] 1,000 [*FV*]

[CPT] [*I*/*Y*]

Result is 3.5392.

[$\times$] 4 [=]

Answer is **14.16%**.

Solution 25

E Chapter 3, Present Value

The number of years until the 1-year-old, 3-year-old, and 6-year-old reach the age of 18 are 17, 15, and 12, respectively.

The number of years until the 1-year-old, 3-year-old, and 6-year-old reach the age of 21 are 20, 18, and 15, respectively.

The present value of the payments matches Choice **E**:

$$Z = X\left[v^{17} + v^{15} + v^{12}\right] + Y\left[v^{20} + v^{18} + v^{15}\right]$$