May 2005 Exam Solutions

Solution 1

E Chapter 6, Level Annuities

The present value of an annuity-immediate is:

$$a_{\overline{n}|} = v^n s_{\overline{n}|} = \frac{s_{\overline{n}|}}{(1+i)^n}$$

By inspection, the expression above is not equal to the expression in Choice **E**.

Solution 2

C Chapter 12, Sinking Fund

The interest payment made at the end of each year is the interest rate of 9% times the loan amount of 10,000:

$$10{,}000 \times 0.09 = 900$$

The sinking fund payments accumulate to 10,000 at the end of 10 years. The sinking fund payment is found below:

$$SFP \times s_{\overline{10}|0.08} = 10{,}000$$

$$SFP \times \frac{1.08^{10} - 1}{0.08} = 10{,}000$$

$$SFP = \frac{10{,}000}{14.4866}$$

$$SFP = 690.2949$$

The total of the payments made over the 10-year period is equal to 10 times the sum of the interest payment and the sinking fund payment:

$$X = 10 \times (690.2949 + 900) = \mathbf{15{,}902.95}$$

The BA-II Plus calculator can be used to solve this question as follows:

10 [*N*] 8 [*I*/*Y*] 10,000[*FV*]

[CPT] [*PMT*] [+/-]

[+] 10,000 [×] 0.09 [=]

[×] 10 [=]

Result is **15,902.95**.

Solution 3

B Chapter 16, Duration

The price of the bond is:

$$\sum_{t>0} PV_0\left(CF_t\right) = \frac{100}{1.20} + \frac{100}{1.20^2} + \frac{1{,}100}{1.20^3} = 789.3519$$

The numerator of the formula for the Macaulay duration is:

$$\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right] = \frac{100 \times 1}{1.20} + \frac{100 \times 2}{1.20^2} + \frac{1{,}100 \times 3}{1.20^3} = 2{,}131.9444$$

The Macaulay duration is:

$$MacDur = \frac{\sum_{t>0}\left[t \times PV_0\left(CF_t\right)\right]}{\sum_{t>0} PV_0\left(CF_t\right)} = \frac{2{,}131.9444}{789.3519} = \mathbf{2.70}$$

Solution 4

A Chapter 6, Level Annuities

Since Seth receives the first *n* payments and Susan receives the next *m* payments, the difference between the present values of their payments is:

$$Xa_{\overline{n|}} - Xv^n a_{\overline{m|}} = X\left[a_{\overline{n|}} - v^n a_{\overline{m|}}\right]$$

This matches Choice **A**.

Solution 5

B Chapter 15, Bonds

The 6-month effective yield is:

$$y = \left(\frac{1{,}000}{624.60}\right)^{\frac{1}{24}} - 1 = 0.01980$$

The price of the bond is:

$$P = Coup \times a_{\overline{n|}y} + Rv^n = 30 \times a_{\overline{20|}0.01980} + \frac{1{,}000}{1.01980^{20}}$$

$$= 30 \times \frac{1 - 1.01980^{-20}}{0.01980} + 675.5668 = 30 \times 16.3825 + 675.5668 = \mathbf{1{,}167.04}$$

The BA-II Plus can be used to answer this question:

1,000 [÷] 624.60 [=] [y^x] 24 [1/*x*] [=] [–] 1 [=] [×] 100 [=] [*I*/*Y*]

20 [*N*] 30 [*PMT*] 1,000 [*FV*]

[CPT] [*PV*]

Result is –1,167.04. Answer is **1,167.04**.

Solution 6

D Chapter 16, Portfolio Duration

The duration of a portfolio is the weighted average duration of its components:

$$MacDur_{Port} = \sum_{j=1}^{k} w_j \times MacDur_j = \frac{980 \times 21.46 + 1,015 \times 12.35 + 1,000 \times 16.67}{980 + 1,015 + 1,000}$$

$$= \frac{50,236.05}{2,995} = \mathbf{16.7733}$$

Solution 7

A Chapter 13, Internal Rate of Return

The time-0 equation of value to be solved is:

$$364.46 = 100 + 200v + 100v^2$$

$$0 = -264.46 + 200v + 100v^2$$

The easiest way to obtain the annual effective interest rate that satisfies the equation above is to use the BA-II Plus calculator:

[CF] CF0 = 264.46 [+/-] [ENTER] ↓

C01 = 200 [ENTER] ↓ ↓

C02 = 100 [ENTER]

[IRR] [CPT]

The result is 10.00.

The answer is **10.00%**.

Solution 8

C Chapter 12, Loans

After the 10th payment, the remaining balance is equal to the present value of the remaining 15 payments:

$$300a_{\overline{15|}}$$

After subtracting out the extra 1,000, the remaining balance is:

$$300a_{\overline{15|}} - 1,000$$

This balance is then paid off with 10 payments, so the time-10 equation of value is:

$$300a_{\overline{15|}} - 1,000 = Pmt \times a_{\overline{10|}}$$

We can now solve for the revised annual payment:

$$300 \times \frac{1 - 1.08^{-15}}{0.08} - 1{,}000 = Pmt \times \frac{1 - 1.08^{-10}}{0.08}$$

$$300 \times 8.5595 - 1{,}000 = Pmt \times 6.7101$$

$$Pmt = \mathbf{233.65}$$

The BA-II Plus calculator can be used to solve this problem as follows:

15 [*N*] 8 [*I/Y*] 300 [*PMT*] [CPT] [*PV*]

[+] 1,000 [=] [*PV*] 10 [*N*]

[CPT] [*PMT*]

Result is **233.65**.

Solution 9

D Chapter 8, Varying Annuities

Using the PIn method, we have:

$P_1 = 100$ $I = 1$ $n = 50$

The present value is:

$$PV_0 = \left(P_1 + \frac{I}{i}\right) a_{\overline{n}|} - \frac{In}{i} v^n = \left(100 + \frac{1}{0.09}\right) \times \frac{1 - (1.09)^{-50}}{0.09} - \frac{1 \times 50}{0.09}(1.09)^{-50}$$

$$= 111.1111 \times 10.9617 - 7.4714$$

$$= \mathbf{1{,}210.49}$$

Using the BA II Plus, we have:

50 [*N*] 9 [*I/Y*] 100 [+] 0.09 [1/*x*] [=] [*PMT*]

50 [÷] 0.09 [=] [+/-] [*FV*]

[CPT] [*PV*]

Result is –1,210.49. Answer is **1,210.49**.

Solution 10

C Chapter 18, Forward Rates

Although the wording of the question is a bit ambiguous, it is meant to refer to the 1-year forward rate, starting in 1 year:

$$1+f_{t-1}=\frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}}$$

$$1+f_1=\frac{(1+s_2)^2}{1+s_1}$$

$$1+f_1=\frac{1.095^2}{1.085}$$

$$f_1=\mathbf{0.1051}$$

Solution 11

C Chapter 15, Callable Bonds

Since this bond is a discount bond, its price-to-worst is found by assuming that the bond is held until maturity. Therefore, the highest price that the investor can pay and be assured of her desired yield is 897.

The bond is called after 20 years for 1,050, and therefore the time-0 equation of value is:

$$897=80\times a_{\overline{20|}}+\frac{1,050}{(1+y)^{20}}$$

The BA-II Plus can be used to find the yield that satisfies the equation of value:

20 [*N*] 897 [+/-] [*PV*] 80 [*PMT*] 1,050 [*FV*]

[CPT] [*I*/*Y*]

Result is 9.2430.

Answer is **9.24%**.

Solution 12

B Chapter 6, Perpetuities

I is only true for level annuities-immediate that pay annually.

II is true.

III is only true for level annuities-immediate.

Solution 13

D Chapter 13, Internal Rate of Return

The time-0 equation of value to be solved is:

$$1,000 + 1,500v = 2,600v^2$$

$$1,000 + 1,500v - 2,600v^2 = 0$$

The easiest way to obtain the annual effective interest rate that satisfies the equation above is to use the BA-II Plus calculator:

[CF] CF0 = 1,000 [ENTER] ↓

C01 = 1,500 [ENTER] ↓ ↓

C02 = 2,600 [+/-] [ENTER]

[IRR] [CPT]

The result is 2.8342%, which is an annual effective interest rate. We must convert it into an interest rate that is convertible semiannually.

[÷] 100 [+] 1 [=] [y^x] 0.5 [–] 1 [=] [×] 2 [=]

Answer is **0.0281**.

Solution 14

E Chapter 8, Varying Annuities

At time 10, the present value of the remaining payments can be found using the PIn method:

Using the PIn method, we have:

$P_1 = 19 \qquad I = -1 \qquad n = 19$

The present value is:

$$PV_{10} = \left(P_1 + \frac{I}{i}\right)a_{\overline{n}|} - \frac{In}{i}v^n = \left(19 - \frac{1}{0.06}\right) \times \frac{1-(1.06)^{-19}}{0.06} + \frac{1\times 19}{0.06}(1.06)^{-19}$$

$$= 2.3333 \times 11.1581 + 104.6624 = 130.6981$$

Discounting the present value found above for 10 years and adding the present value of the first 10 payments gives us the present value of the annuity-immediate:

$$130.6981v^{10} + 20a_{\overline{10}|} = \frac{130.6981}{1.06^{10}} + 20 \times \frac{1-1.06^{-10}}{0.06} = \mathbf{220.18}$$

We can use the BA II Plus to answer this question:

19 [*N*] 6 [*I*/*Y*] 19 [–] 0.06 [1/*x*] [=] [*PMT*]

19 [÷] 0.06 [=][*FV*]

[CPT] [*PV*]

(Result is –130.6981)

[+/-] [*FV*]

10 [*N*] 20 [*PMT*]

[CPT] [*PV*]

Result is –220.18. Answer is **220.18**.

Solution 15

D Chapter 17, Dedication

The present value of the asset cash flows is equal to the present value of the liability cash flows, so the cost of the bonds is equal to the present value of the liability cash flows:

$$\frac{10{,}000}{1.05}+\frac{10{,}000}{1.05^2}=\mathbf{18{,}594.10}$$

Solution 16

A Chapter 13, Dollar-Weighted Rate of Return

To find the income we treat the final balance as a withdrawal and we treat the initial balance as a deposit:

$$\text{Income}=\text{Withdrawals}-\text{Deposits}=200+500+1{,}560-1{,}000-1{,}000=260$$

The exposure of the fund to interest is:

$$\text{Fund exposure}=\sum(\text{Net deposit})(\text{Time deposit is in the fund})$$
$$=1{,}000+1{,}000\times\frac{8}{12}-200\times\frac{6}{12}-500\times\frac{4}{12}=1{,}400$$

The simple interest approximation to the dollar-weighted yield is:

$$i=\frac{\text{Income}}{\text{Fund exposure}}=\frac{260}{1{,}400}=\mathbf{0.1857}$$

Solution 17

B Chapter 8, Varying Annuities

We can use the formula for the present value of an increasing perpetuity-immediate to solve for i:

$$PV_0=\frac{P_1}{i}+\frac{I}{i^2}$$

$$46{,}530=\frac{200}{i}+\frac{50}{i^2}$$

$$46{,}530i^2=200i+50$$

$$46{,}530i^2-200i-50=0$$

The quadratic formula gives us two solutions for i, and we use the positive one:

$$i = \frac{200 \pm \sqrt{200^2 + 4 \times 46{,}530 \times 50}}{2 \times 46{,}530}$$

$$i = \mathbf{0.0350}$$

Solution 18

E Chapter 3, Present Value

Let P be the purchase price. The two offers have the same present value:

$$0.90P(1.08)^{-2/12} = P\left(1 - \frac{X}{100}\right)$$

$$0.90 = \left(1 - \frac{X}{100}\right)(1.08)^{1/6}$$

The final equation above matches Choice **E**.

Solution 19

C Chapter 3, Interest Rate Conversions

The monthly effective interest rate can be converted to the monthly effective discount rate:

$$\frac{d^{(12)}}{12} = \frac{\frac{i^{(12)}}{12}}{1 + \frac{i^{(12)}}{12}} = \frac{\frac{0.189}{12}}{1 + \frac{0.189}{12}} = 0.01551$$

The nominal annual discount rate is the monthly effective rate times 12:

$$d^{(12)} = 0.01551 \times 12 = \mathbf{0.1861}$$

Solution 20

A Chapter 8, Varying Annuities

There are two funds, one earning 12% and one earning 8%. The amount of each level deposit is denoted by X. The deposits into the funds are described below:

Time	12%	8%
0	X	
1	X	$X \times 0.12$
2	X	$2X \times 0.12$
…	…	…
9	X	$9X \times 0.12$
10		$10X \times 0.12$

Since the interest from the 12% fund is paid into the 8% fund, the value of the 12% fund at the end of 10 years is $10X$. The accumulated value of the 8% fund can be found using an increasing annuity immediate:

$$10X + (Is)_{\overline{10}|0.08} \times 0.12X = 10{,}000$$

$$10X + \frac{\ddot{s}_{\overline{10}|0.08} - 10}{0.08} \times 0.12X = 10{,}000$$

$$10X + \frac{\frac{1.08^{10} - 1}{0.08/1.08} - 10}{0.08} \times 0.12X = 10{,}000$$

$$10X + \frac{15.6455 - 10}{0.08} \times 0.12X = 10{,}000$$

$$10X + 70.5686 \times 0.12X = 10{,}000$$

$$X = \mathbf{541.47}$$

We can use the BA II Plus to obtain the value of $(Is)_{\overline{10}|0.08}$:

10 [*N*] 8 [*I*/*Y*] 1 [*PMT*] [CPT] [*FV*] [×] 1.08 [+/-]

[–] 10 [=] [÷] 0.08 [=]

Result is 70.5686.

Solution 21

D Chapter 7, Level Annuities

Let *P* be the purchase price. The equation of value at time 0 is:

$$P = \frac{P}{10}\ddot{a}_{\overline{12}|j}$$

$$1 = 0.1 \times \ddot{a}_{\overline{12}|j}$$

We can use the BA-II Plus calculator to solve for the monthly effective interest rate. Once we have the monthly effective interest rate, we convert it into the annual effective interest rate:

[2nd] [BGN] [2nd] [SET] [2nd] [QUIT]

12 [*N*] 1 [*PV*] 0.1 [+/-] [*PMT*] [CPT] [*I*/*Y*]

(Result is 3.5032)

[÷] 100 [+] 1 [=] [y^x] 12 [=] [–] 1 [=]

Answer is **0.5116**.

Solution 22

B Short Selling

The margin is the margin requirement times the value of the stock:

$$0.80 \times 50 = 40$$

On January 1, 2005, the profit to Karen is the price at which she sold the stock minus the price at which she covers the short sale plus the margin interest minus the dividend on the stock:

$$50 - X + 4 - 2 = 52 - X$$

The percentage return is equal to the January 1, 2005 profit over the margin requirement:

$$\frac{52 - X}{40} = 0.20$$

$$X = \mathbf{44}$$

Solution 23

D Chapter 14, Common Stock

We can use the dividend discount growth model to find the return on the stock:

$$PV_0 = \frac{Div_1}{i - g}$$

$$75 = \frac{6}{i - 0.03}$$

$$i = \mathbf{0.11}$$

Solution 24

E Chapter 6, Level Annuities

The accumulated value of the annuity-immediate at time $(n+1)$ is equal to the accumulated value of an annuity-due:

$$s_{\overline{n|}} \times (1+i) = 13.776$$

$$\ddot{s}_{\overline{n|}} = 13.776$$

$$\frac{(1+i)^n - 1}{d} = 13.776$$

$$\frac{2.476 - 1}{d} = 13.776$$

$$d = 0.1071$$

We can use d to find the value of n:

$$(1+i)^n = 2.476$$

$$(1-d)^{-n} = 2.476$$

$$(1-0.1071)^{-n} = 2.476$$

$$-n \times \ln(0.8929) = \ln(2.476)$$

$$n = \mathbf{8.00}$$

Solution 25

A Chapter 12, Loans

The payment of 20 is exactly equal to the interest on the loan each quarter:

$$500 \times \frac{0.16}{4} = 20$$

Since the payment does not exceed the interest, the amount of principal paid down is zero for each payment:

$$Prn_t = Pmt_t - Int_t = 20 - 20 = \mathbf{0}$$